Mixture Problems
Past, Present, and Future

AMATYC 2010 Boston

Dr. David Graser

Yavapai College

Prescott, Arizona

David_Graser@yc.edu / www.pblpathways.com

Concerned Citizen Commuters Association
108 Manhattan Ave.
Suite 1304
New York City, NY 09756

Independent Mathematical Contractors
00 Anystreet
Anytown, Anystate 00000

Dear IMC:

My advocacy group is concerned with reducing carbon emissions via all possible methods. In the past we have focused on building new subway lines in New York City and increasing the ridership on those lines. We are now starting a campaign to increase the mileage in cars for people who have no choice but to drive their cars to work. Our research has focused on bettering the mileage in existing cars through a modification of the fuel blend being used. Our experience has been that 88.5 octane gasoline leads to better mileage.

Several different types of gasoline are available across the country. These gasolines are distinguished by their octane levels.

Name	Octane	Average Price on 3/3
Regular	87	$3.049
Mid	89	$3.180
Premium	91	$3.363

The term “octane” has more than one definition, but the one used by the State of Minnesota is fairly useful:

> Octane is a measure of a fuel’s tendency to knock or ping when it is mixed with air and burned in the cylinder of an engine. This octane rating is not based on the amount of chemical octane in the gasoline. The rating is called octane because the gasoline’s ability to prevent engine knock has been rated against the performance of pure hydrocarbon octane, which has a rating of 100. Gasoline, which is made from a blend of many other hydrocarbons, may have a higher or lower rating, depending on how its anti-knock performance compares to the performance of pure hydrocarbon octane.

You can think of this octane as a percentage of pure octane in the gasoline. In other words, 87 octane gas is 87% pure octane, 89 octane gas is 89% pure octane, ect. Across the country you may find slight differences in octane, but for the most part these fuel grades are reflective of what is available from refiners.

To make your decision, you’ll need to develop and solve a system of equations under some assumptions.

1. Instead of using the average prices on 3/3/08 above, use the octane levels and prices from your favorite gas station.

2. Assume that you are filling your own empty tank. If you do not own a car, you can assume a tank with a capacity of 16 gallons.

The question I would like you to answer is quite simple: By blending the three available grades of gas, how many gallons of each type of gasoline must be blended to obtain an 88.5 octane mixture that is as cheap as possible?

I assume that you will develop and solve a system of equations based on the information you find in your local gas station. You will need to create a system of two equations with three variables. One of these equations will insure that the total amount of gas will fill your tank. The other equation will insure that your gas mixture is 88.5 octane. Make sure you explain how you got your equations and the steps you followed to solve these equations.

Since I am not a mathematically inclined person (college algebra was over 10 years ago), I would like you to document your results in a technical memo. Your scientific expert (your instructor) has made resources available for you to help you produce this document.

I look forward to your technical memo on this matter. A scientific expert is available to answer any questions that you might have in the course of your investigations. This expert will not be available to assist on this project over the weekend before it is due. You should plan on consulting with this expert as soon as possible.

Sincerely yours,

Jack B. Green

City of Bad Water
100 Main Street
Bad Water, NV 89422

Independent Mathematical Contractors
Any College
1 Your Street
City, State 00000

Dear IMC:

The city I work for is currently undergoing a project to reduce the level of arsenic in its drinking water. Our city receives its drinking water from three different wells located around the city. Each well contains arsenic at varying levels. The Environmental Protection Agency gave our city an exemption for several years in complying with EPA rules. However, we are required to submit a plan to them in 6 months regarding how we will reduce the level of arsenic in the city's drinking water to an acceptable level.

Arsenic is an element in the periodic table that occurs naturally in the earth's crust. It also can enter the drinking water supply through agricultural and industrial practices. When ingested, arsenic can cause thickening and discoloration of the skin, stomach pain, nausea, vomiting, diarrhea, numbness in the extremities, paralysis and blindness. As if that was not enough, arsenic has been linked to several cancers including cancer of the bladder, lungs, skin, kidney, nasal passages, liver and the prostrate.

Under previous rules, my city was well within the acceptable arsenic standard which was 50 parts per billion (ppb). However, these rules changed and required compliance with a new standard by January 23, 2006. This standard was set at 10 ppb and only one of our wells was able to meet that standard. At the time, we were able to use this well to supply the drinking water for the entire city. Growth has necessitated that we use the other wells (that do not meet the standard).

The table below shows the three wells, their respective levels of arsenic and their capacities.

Well	**Arsenic Level (ppb)**	**Capacity (millions of liters per day)**
1	5	1.5
2	8 + F	2.0
3	10 + L	1.0

In this table, F is the number of letter in your first name and L is the number of letters in your last name. The city has an average daily demand of $3.5+0.01\cdot M$ million liters that must be supplied from these three wells. M is the number of letters in your middle name.

To meet this demand, we plan to blend water from these three wells. This is done by piping the water to a central location and then combining different amounts from each well together to get a mixture that contains 8 ppb of arsenic. To help us write the plan for the EPA, we would like to know how much water we must pump daily from each of the wells.

Please document your work in a technical memo. This document must contain enough detail so that we could modify your calculations should any of the numbers above change. This means that numbers alone will not be sufficient. We need to see your calculations and understand the steps you followed to solve the problem outlined above.

Sincerely,

Mortimer Bruster
Director of Utilities
City of Bad Water, Nevada

City of Bad Water
100 Main Street
Bad Water, NV 89422

Independent Mathematical Contractors
00 Anystreet
Anytown, Anystate 00000

Dear IMC:

I recently contacted you regarding the blending of water from three different wells to lower the amount of arsenic. In examining your solutions to our problem, it occurred to us that there might be other contaminants in our drinking water. An analysis of our water found a problem with another contaminant, selenium, in addition to the arsenic you already are familiar with.

Selenium is a metal that is found naturally in ore deposits. It is used in the manufacture of electronic s and photocopiers as well as in other industrial activities. While selenium is an essential nutrient at low levels, higher levels can cause damage if a person is exposed over a long period of time. Long term damage, such as that incurred by drinking water with high levels of selenium, can lead to kidney and liver damage.

The EPA has established a level of 0.05 part per million as the maximum allowable level in drinking water. Recall that the level for arsenic is 10 parts per billion. To insure a safety factor of 20%, we would like the blended water to have a level of selenium no higher than 0.04 parts per million and a level of arsenic no higher than 8 parts per billion.

The table below shows the three wells, their respective levels of arsenic and selenium.

Well	**Arsenic Level (ppb)**	**Selenium Level (ppm)**
1	5	0.03
2	8 + F	0.02
3	10 + L	0.05

In this table, F is the number of letter in your first name and L is the number of letters in your last name. The city has an average daily demand of $3.5+0.01\cdot M$ million liters that must be supplied from these three wells. M is the number of letters in your middle name. We need to make sure that we have at least this capacity to supply drinking water to our customers. We are no longer worried about the capacities from each well. You earlier research has convinced us that it is fairly cost effective to replace pumps to increase well capacity.

To meet this demand, we plan to blend water from these three wells. This is done by piping the water to a central location and then combining different amounts from each well together to get a mixture that no more than 8 ppb of arsenic and 8 ppm of selenium.

Our major constraint in this project is cost. The cost of pumping and transporting water to the central location varies. Since well 1 is farthest away from the central location, it costs the most to produce this water. Well 3 is the closest so it costs the least to produce the water. The table below gives the cost per thousand liters to produce water from each well.

Well	Cost per Thousand Liters (Dollars)
1	1.00
2	0.80
3	0.70

To help us write the plan for the EPA, we would like to know how much water we must pump daily from each of the wells. To insure the most efficient method possible, we want to do this at as low a cost as possible.

Please document your work in a technical memo. This document must contain enough detail so that we could modify your calculations should any of the numbers above change. This means that numbers alone will not be sufficient. We need to see your calculations and understand the steps you followed to solve the problem outlined above.

Sincerely,

Mortimer Bruster
Director of Utilities
City of Bad Water, Nevada