Baye’s Theorem as presented in the textbook is fairly complicated. I would not want to memorize it so I am guessing that you would not want to also. An alternative approach is to utilize the definition of conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If you multiply both sides by P(B), you get

$$P(A|B)P(B) = P(A \cap B)$$

Utilizing this kind of thinking we could also create a definition for P(B|A):

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

By multiplying both sides by P(A), we get

$$P(B|A)P(A) = P(B \cap A)$$

Since $A \cap B = B \cap A$, we know that $P(A \cap B) = P(B \cap A)$. Putting this all together gives

$$P(A|B)P(B) = P(A \cap B) = P(B|A)P(A)$$

If I eliminate the middle part we get a kinder, gentler version of Baye’s Theorem

$$\boxed{P(A|B)P(B) = P(B|A)P(A)}$$

This relates “A given B” to “B given A” and allows us to solve some very interesting problems like the example below.

Machine Operators

Based on past experience, a company knows that an experienced machine operator (one or more years of experience) will produce a defective item 1% of the time. Operators with some experience (up to one year) have a 2.5% defect rate, and new operators have a 6% defect rate. At any one time, the company has 60% experienced operators, 30% with some experience, and 10% new operators. Find the probability that a particular defective item was produced by a new operator.

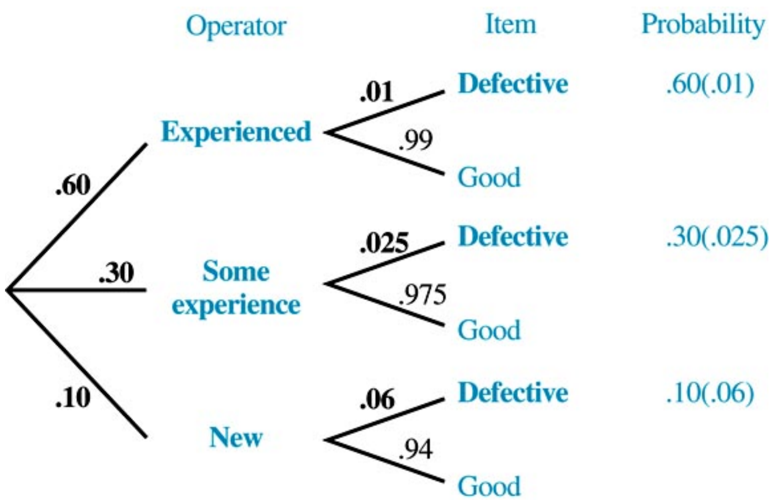
The first thing we need to do is organize the information we have been given. So let’s create some events to work with.

D = “operator produces a defective item”
E = “experienced operator”
S = “operator with some experience”
N = “new operator”

With these definitions, the information in the problem statement can be written as

$$P(E) = 0.60 \qquad P(D \mid E) = 0.01$$
$$P(S) = 0.30 \qquad P(D \mid S) = 0.025$$
$$P(N) = 0.10 \qquad P(D \mid N) = 0.06$$

The tree diagram below reflects this information.



If we want to know the probability that an experience operator produces a defective item, we are interested in the event $E \cap D$. These probabilities lie along the top branch so

$$P(E \cap D) = P(D \mid E)P(E) = (.60)(.01) = .006$$

What would P(N|D) mean? This means that if a defective item is made, what is the likelihood that a new operator produced it? The appropriate form of Baye's Theorem is

$$P(N \mid D)P(D) = P(D \mid N)P(N)$$

Solving for P(N|D) yields

$$P(N \mid D) = \frac{P(D \mid N)P(N)}{P(D)}$$

Both factors in the numerator are branches in the tree. The denominator is simply the sum of the branches leading to defective or

$$P(N \mid D) = \frac{(.06)(.10)}{(.01)(.60) + (.025)(.30) + (.06)(.10)} \approx .31$$

The key to using Baye's Theorem is to write the proper tree and appropriate rule based on the events in the problem.