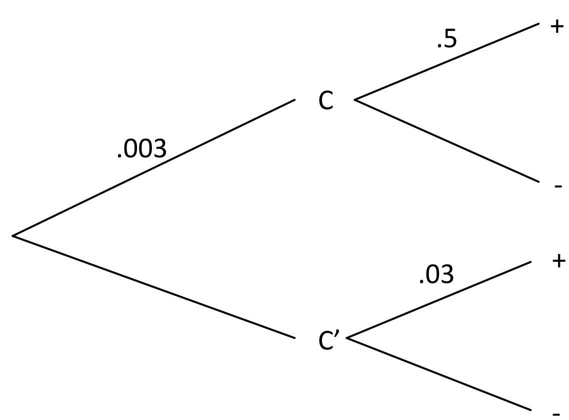
The probability of colorectal cancer can be given as .3%. If a person has colorectal cancer, the probability that the hemoccult test is positive is 50%. If a person does not have colorectal cancer, the probability that he still tests positive is 3%. What is the probability that a person who tests negative does not have colorectal cancer?

To solve this problem, we'll draw and label an appropriate tree diagram. Then we'll apply Bayes' Rule to the problem.

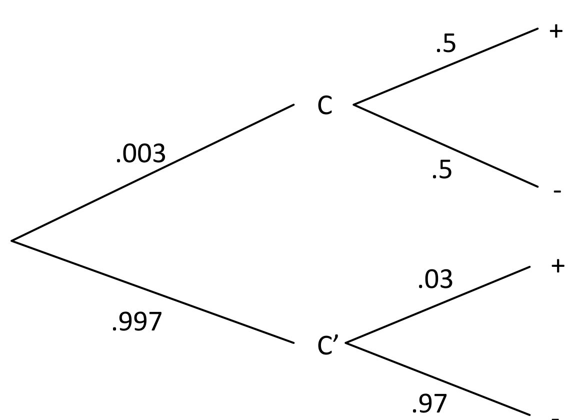
Draw a tree diagram: Look at the information given in the problem. If C is the event "person has colorectal cancer" and + is the event "the hemoccult test is positive", we know that

$$P(C) = .003 \quad P(+ \mid C) = .5 \quad P(+ \mid C') = .03$$

This suggests the following tree diagram:



Knowing that the sum of the probabilities from one point on the tree should add to 1, we can finish the tree diagram as follows:



Apply Bayes' Rule: The probability we are looking for is $P(C'|-)$. Notice that the tree diagram has $P(-|C')$, but not the reverse conditional probability that we are looking for. This is a sign we need to use Bayes' Rule. Let's find the appropriate form of Bayes' Rule. The definition of conditional probability applied to these events tells us

$$P(C'|-) = \frac{P(C' \cap -)}{P(-)}$$
$$P(-|C') = \frac{P(- \cap C')}{P(C')}$$

Solving each of these for the intersection yields

$$P(C'|-)P(-) = P(C' \cap -)$$
$$P(-|C')P(C') = P(- \cap C')$$

Since the intersection on the right hand side is the same in each equation, we know the left hand sides must be equal.

$$P(C'|-)P(-) = P(-|C')P(C')$$

Solving for $P(C'|-)$ gives

$$P(C'|-) = \frac{P(-|C')P(C')}{P(-)}$$

This is Bayes' Rule for this problem. Now we are ready to use the tree diagram. $P(-|C')$ and $P(C')$ are both labeled on the tree diagram. We can calculate $P(-)$ by following the branches on the tree diagram (multiply) that lead to a negative result, and then summing up the products from these branches.

$$P(-) = (.003)(.5) + (.997)(.97) = .96859$$

Putting these values into Bayes' Rule gives

$$P(C'|-) = \frac{(.97)(.997)}{.96859} \approx .9985$$

This means that is you test negative, the likelihood that you do not have colorectal cancer is 99.85%. The test is quite good at screening that you do not have the disease.