

## Revenue Functions

As discussed before, businesses convert inputs to outputs. This process costs businesses money which is described by the total cost function  $C(x)$ . To compensate for these costs, businesses receive money for selling goods or services. The amount received depends on the inputs to the businesses and is called revenue. The revenue function is denoted by the letter  $R$  and is a function of the company's inputs. Typically revenue is described in the short term where at least one of the inputs is fixed. If all but one of the inputs are fixed, the revenue can be denoted by a function of one variable  $R(x)$ .

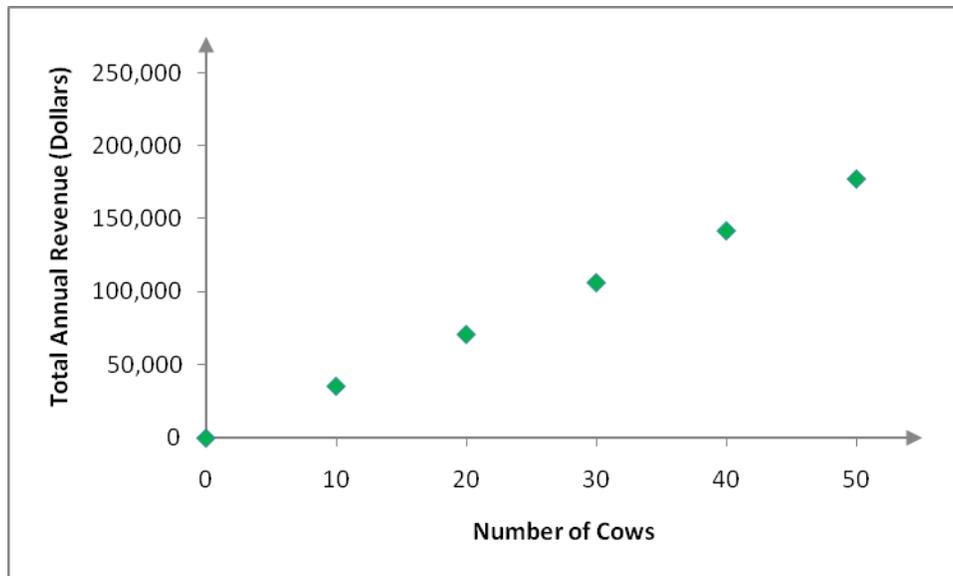
### Example – Dairy Revenue

A dairy can increase its milk production and thus its revenue in several ways. By changing the feed or administering certain hormones, the amount of milk produced can be increased and lead to higher total revenue. Revenue can also be changed by increasing or decreasing the number of dairy cows. More cows can produce more milk thus leading to higher revenue. In this example, we are interested in varying the number of cows  $x$  and seeing how the total annual revenue  $R(x)$  changes. All other inputs to the business will be held constant in the short run.

The table below describes the level of total annual revenue for various numbers of dairy cows.

Number of Dairy Cows	Total Annual Revenue (Dollars)
0	0
10	35,470
20	70,940
30	106,410
40	141,880
50	177,350

Let's visualize this data by graphing the total annual revenue as a function of the number of dairy cows. By indicating that this is a function of the number of dairy cows, we know that the number of dairy cows will be graphed horizontally and the total annual cost will be graphed vertically.



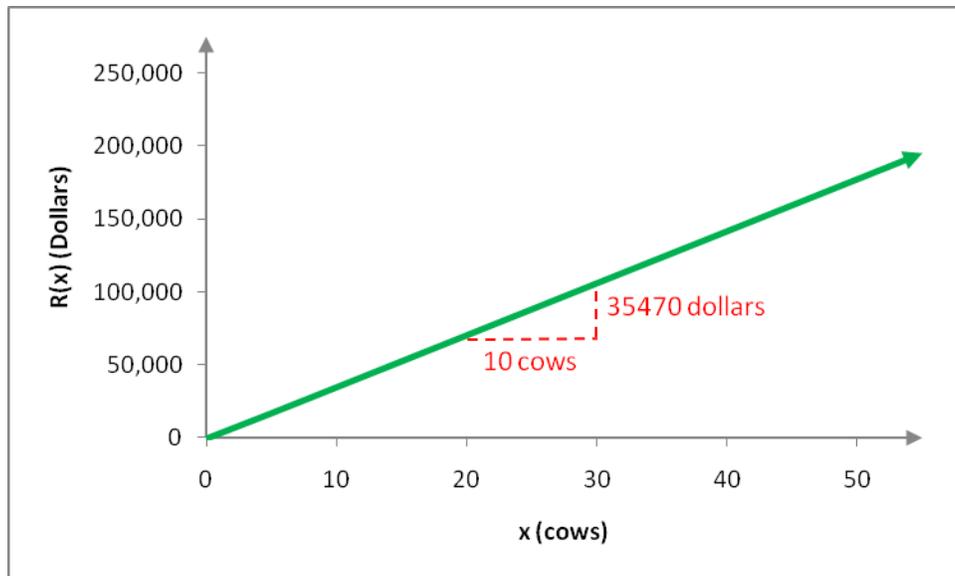
Like the total annual cost, the total annual revenue appears to follow a straight line.

Ordered Pairs	Slope
(0, 0)	$\frac{35470 - 0}{10 - 0} = 3547$
(10, 35470)	
(20, 70940)	$\frac{106410 - 70940}{30 - 20} = 3547$
(30, 106410)	
(40, 141880)	$\frac{177350 - 141880}{50 - 40} = 3547$
(50, 177350)	

The slope between any pair of points on the line is 3547. This indicates that the ordered pairs all lie along a line that has a slope of 3547. Since the vertical intercept of this line is the point (0,0), we can use the slope-intercept form of a line to write

$$R(x) = 3547x$$

The graph of this function mirrors the trend of the data in the scatter plot above.



As with the cost, we can think of the slope of the revenue function as a ratio,

$$\frac{\text{Change in Revenue}}{\text{Change in Dairy Cows}} = \frac{35470 \text{ dollars}}{10 \text{ cows}} = \frac{3547 \text{ dollars}}{1 \text{ cows}}.$$

An increase in the number of dairy cows by 10 leads to an increase in revenue of \$35,470. This means that each additional dairy cow adds \$3547 to the revenue. The amount of revenue added when the input increases by one unit is called the marginal revenue.