Break Even Point

When the total revenue and total costs are equal, the business is at the break-even point. This point can be found graphically by locating the point of intersection on the total cost and total revenue graphs. The same point can be found algebraically by setting the total cost function equal to the total revenue function. Solving for the variable gives the value of the input at the break-even point. The cost or revenue at that value is found by substituting it into either the total cost or revenue function.
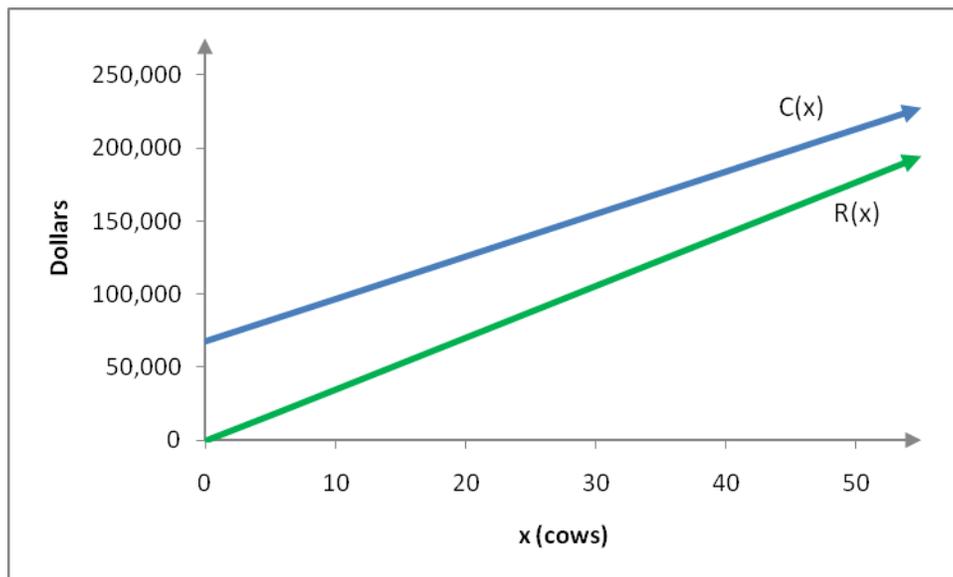
Example – Dairy Break-Even Point

In earlier examples, we found the total cost $C(x)$ and the total revenue $R(x)$ for a dairy as a function of the number of cows, x. The total cost function,

$$C(x) = 2890.8x + 68688,$$

and the revenue function,

$$R(x) = 3547x,$$

are graphed below.

In the window shown, the two graphs do not intersect. However, since the revenue function is steeper than the cost function they will eventually intersect. We could experiment with the graph's window to find where the graphs intersect. But it is easier to find the point of intersection algebraically and then check it on the graph.

Start by setting the total cost function equal to the total revenue function,

$$2890.8x + 68688 = 3547x.$$
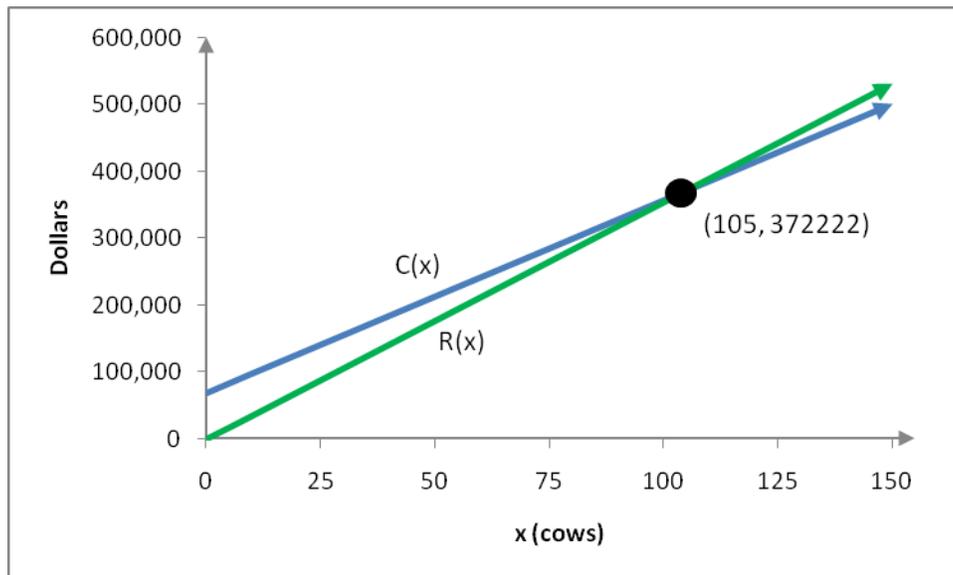
Subtract $2890.8x$ from both sides to yield

$$68688 = 656.2x .$$

Divide both sides by 656.2,

$$\frac{68688}{656.2} = x$$

This is approximately 104.68.

Now let's extend the window of the graph to include the break-even point.

Since x represents the number of dairy cows, we round the x-value to the nearest integer or 105. At this level,

$$C(105) = 2890.8(105) + 68688 = 372,222$$
$$R(105) = 3547(105) = 372,435$$

At the nearest integer, the revenue and cost are not exactly equal. They differ by $213. What about at the next nearest integer, 104?

$$C(104) = 2890.8(104) + 68688 = 369,331.2$$
$$R(104) = 3547(104) = 368,888$$

The total cost and total revenue differ by $443.2.

For the number of dairy cows that make sense, the revenue does not match the cost exactly. In this case, we'll round up to 105 since the total revenue and total cost are closer at this value.