

## Radioactive Dating

Radioactive decay is a natural process by which an unstable atomic nucleus spontaneously loses energy. In losing this energy, the atom transforms to a new type of atom. For instance, a Carbon-14 (C-14) atom will randomly emit radiation to become a Nitrogen-14 (N-14) atom (N-14).

Although the energy is emitted randomly, it does so at a predictable rate. For C-14, half of the material will decay to N-14 every 5730 years. Other unstable radioactive materials will decay faster or slower depending on this half life. In general, the half life describes the length of time it takes for a substance to decay to half its original amount.

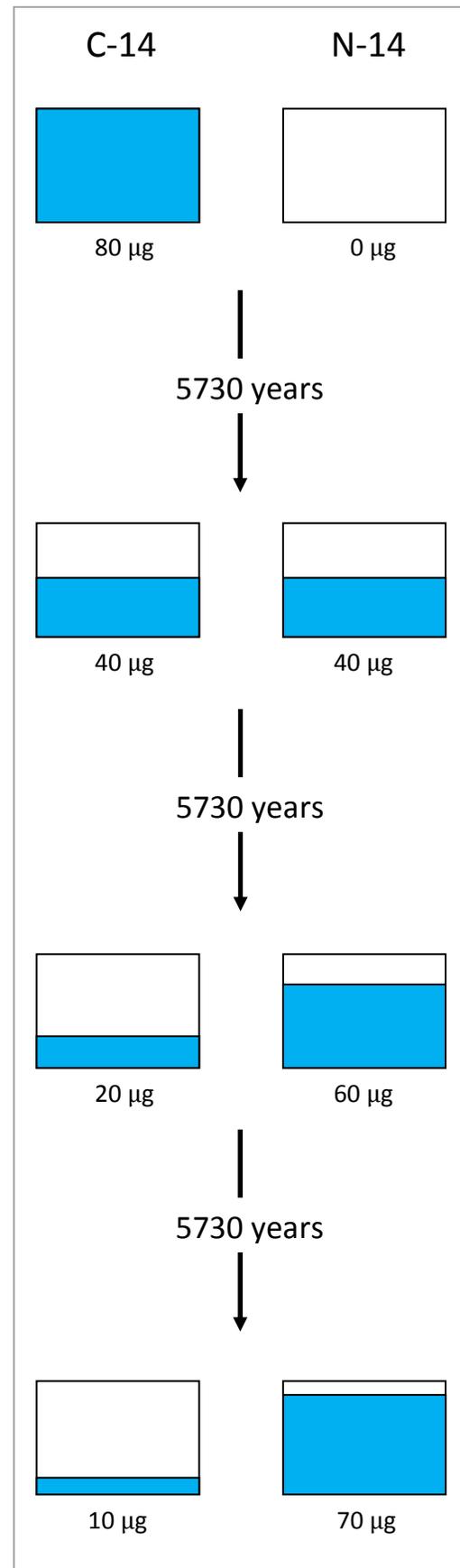
In the diagram shown to the right, a sample initially contains 80 micrograms or 80  $\mu\text{g}$  of C-14 and no N-14. Over time, the C-14 will change into N-14 at a predictable rate.

After 5730 years, half of the C-14 or 40  $\mu\text{g}$  remains. The other 40  $\mu\text{g}$  has decayed into N-14. After another 5730 years, another half of the C-14 or 20  $\mu\text{g}$  remains. The amount of C-14 that transformed into N-14 leaves the total amount of N-14 at 60  $\mu\text{g}$ . This process of halving the amount of C-14 and adding it to N-14 continues indefinitely. At some point there is virtually no C-14 left and all of the C-14 has become N-14.

By comparing the amount of C-14 in the sample to the original amount in the sample, we can determine the approximate age of the sample. The amount of C-14 in the sample,  $y$ , is

$$y = y_0 e^{kt}$$

where  $y_0$  is the original amount of C-14,  $k$  is the decay constant for C-14 and  $t$  is the age of the sample. In the



case of the sample above, we know that the original amount of C-14 is  $y_0 = 80$ . This means that we can write

$$y = 80 e^{kt}$$

To find the decay constant, we need to use the half life for C-14 of 5730 years. At this time we know that 40  $\mu\text{g}$  of C-14 remains. Think of this data as the ordered pair  $(t, y) = (5.730, 40)$  where the time has been written in thousands of years. Any units for time can be used as long as we consistently use those units throughout the problem. Substituting these values into the equation gives

$$40 = 80 e^{k(5.730)}$$

To solve for k, we'll isolate the exponential function and divide by 80. This yields

$$0.5 = e^{k(5.730)}$$

Convert this exponential equation to logarithm form:

$$k(5.730) = \ln(0.5)$$

Divide both sides of the equation by 5.730 to give

$$k = \frac{\ln(0.5)}{5.730} \approx -0.12097$$

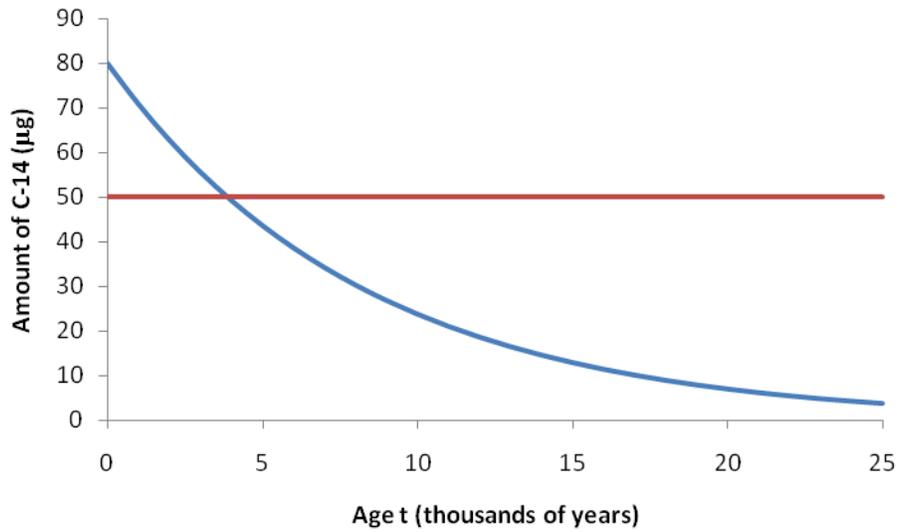
Using this value in the original equation gives us a relationship between the amount of C-14 in the sample  $y$  and the age of the sample  $t$  in thousands of years:

$$y = 80 e^{-0.12097t}$$

To find the age of a sample containing 80  $\mu\text{g}$  of C-14 initially, we need to substitute the amount of C-14 measured in the sample in place of  $y$ . For instance, to determine the age of a sample containing 50  $\mu\text{g}$  of C-14 we would solve

$$50 = 80 e^{-0.12097t}$$

for  $t$  using algebra (convert to logarithm form) or using the method of intersection on a graph. Ideally we should do both and show that the solutions are the same. A graph of this equation shows the decay of the amount of C-14 as the age of the sample increases.



By either method, the age of the sample is approximately 4 thousand years old. On your project, your goal should be to solve the equation algebraically and to verify that answer with a graph like the one above.