The complement of an event E is all of the outcomes in the sample space that are not in the event E. The complement of an event E is represented by the symbols E'. In discussing the complement, we often refer to it as the outcomes not in E. Since the event E and the event not in E combine to give the entire sample space S,

$$P(E)+P(E')=P(S)$$

The likelihood of an outcomes in the sample space occurring is certain, so we simplify this to

$$P(E)+P(E')=1$$

This leads us to a convenient relationship for determining the likelihood that an outcome in the complement will occur.

Probability of the Complement of an Event

The probability that an outcome in the complement E' will occur is

$$P(E')=1-P(E)$$

In other words, the probability that an event will not occur is 1 minus the probability that it will occur.

Example 3 Probability of the Complement of an Event

In Example 1, we calculated the probability that a smartphone user will use some amount of data in a month. The probability for each outcome and the corresponding probability is listed in the table below

Outcome	Approximate Probability
Less than 200 MB	0.345
200 MB up to, but not including 500 MB	0.203
500 MB up to, but not including 1 GB	0.113
1 GB up to, but not including 2 GB	0.087
2 GB or more	0.253

Let E be the event "1GB or more".

a. Describe the complement of E, E'.

Solution The event E contains the one outcome, "2 GB or more". The complement must contain all of the other outcomes that are in the sample space. We can list those outcomes by writin

$$E' = \left\{ \begin{array}{l} \text{"Less than 200 MB", "200 MB up to, but not including 500 MB",} \\ \text{"500 MB up to, but not including 1 GB", "1 GB up to, but not including 2 GB}\end{array} \right\}$$

This is equivalent to saying that less than 2 GB is used.

b. Find the probability that less than 2 GB of data is used.

Solution This event contains four outcomes in the sample space. We could add the probabilities of these outcomes to find likelihood of using less than 2 GB of data. However, it is easier to think of this event as the complement of E and compute

$$P(E') = 1 - P(E)$$
$$\approx 1 - 0.253$$
$$\approx 0.747$$

Since the decimal used for $P(E)$ is approximate, so is the value

calculated for $P(E')$.

In Example 3, it is easier to thin of the event in question as the complement of another event that is simpler to calculate. If the event consists of many outcomes in the sample space, the complement may contain a lot fewer outcomes. This may make it easier to calculate. In the case above, it is easier to use complements than compute

$$P(\text{less than 2 GB}) \approx 0.345 + 0.203 + 0.113 + 0.087$$
$$\approx 0.748$$

The slight difference between the numbers is due to rounding when the relative frequencies were computed in Example 1.