How do you find the likelihood of a certain type of license plate?

Problems involving personal identification numbers, passwords, and license plates usually require the use of the Multiplication Principle. You might be tempted to use permutations, but letters and numbers may generally be repeated in these types of problems. In fact, two of the most common ATM PINs are 1111 and 0000. In permutations, the repetition of numbers is not allowed.

Example 1   License Plate Numbers

Many states offer license plates which a person may choose the letters and numbers on it. Typically, people choose words or phrases with some meaning for them. Sometimes randomly generated license plates have sequences of numbers or letters that might be offensive or have a separate meaning. If the license plates in a particular state consist of three letters followed by three numbers, what is the probability that a randomly generated plate ends in the numbers "911".

Solution Suppose the experiments is choosing a license plate randomly that consists of three letters followed by three numbers. In Section 8.1, we found the total number of possible license plate with the Multiplication Principle,

$$\underbrace{26}_{\substack{\text{first} \\ \text{letter}}} \cdot \underbrace{26}_{\substack{\text{second} \\ \text{letter}}} \cdot \underbrace{26}_{\substack{\text{third} \\ \text{letter}}} \cdot \underbrace{10}_{\substack{\text{first} \\ \text{number}}} \cdot \underbrace{10}_{\substack{\text{second} \\ \text{number}}} \cdot \underbrace{10}_{\substack{\text{third} \\ \text{number}}} = 17,576,000$$

To find the probability that a license plate ends with "911", we also need to know how many license plates end with "911". Applying the Multiplication Principle again gives

$$\underbrace{26}_{\substack{\text{first} \\ \text{letter}}} \cdot \underbrace{26}_{\substack{\text{second} \\ \text{letter}}} \cdot \underbrace{26}_{\substack{\text{third} \\ \text{letter}}} \cdot \underbrace{1}_{\substack{\text{first} \\ \text{number}}} \cdot \underbrace{1}_{\substack{\text{second} \\ \text{number}}} \cdot \underbrace{1}_{\substack{\text{third} \\ \text{number}}} = 17,576$$

There is only 1 way to make the last three choices since the numbers must be "911". The probability of a license plate ending with "911" is

$$P(\text{plate ends with "911"}) = \frac{n(\text{plates ending with "911"})}{n(\text{plates with proper format})}$$

$$= \frac{17{,}576}{17{,}576{,}000}$$

$$= \frac{1}{1000}$$

In problems like this example, the Multiplication Principle is used since letters and numbers on the plate may be repeated.