Question 2: How do you find the likelihood of a particular committee?

In Section 8.2, we counted the number of ways to create an executive leadership team. Since this was simply a grouping of executives and each team member was different, combinations were used to count the number of ways to put together the team.

Example 2 Team Selection

A large nonprofit corporation wishes to form an executive leadership team from a group of 6 male executives and 8 female executives. The team will have 6 members. If the team members are selected randomly, find the probability of each team described below.

a. The team has equal numbers of men and women.

Solution The experiment for this example is the random selection of a team of six. A team with equal numbers of men and women will have three men and three women. Since rearranging team members leads to an identical team, combinations are used to count the teams. The number of six person teams selected from the fourteen executives is

$$C(14,6)=\frac{14!}{(14-6)!\,6!}=3003 \text{ teams}$$

The number of teams with equal numbers of men and women is calculated using the Multiplication Principle. The choices are choosing the men and choosing the women. This gives the number of teams,

Choose 3 members from 6 men — Choose 3 members from 8 women

$$\underbrace{C(6,3)}_{\substack{\text{Choose}\\ \text{3 men}}}\cdot\underbrace{C(8,3)}_{\substack{\text{Choose}\\ \text{3 women}}}=20\cdot 56=1120 \text{ teams}$$

The probability of randomly selecting a team with equal numbers of men and women is

$$P(\text{Team with 3 men and 3 women}) = \frac{n(\text{Teams with 3 men and 3 women})}{n(\text{Teams with 6 members})}$$
$$= \frac{1120}{3003}$$
$$\approx 0.373$$

b. The team has more than four women.

Solution Teams with more than four women must have five women and one man or six women and no men. The number of teams with five women and one is

$$\underbrace{C(8,5)}_{\text{Choose 5 women}} \cdot \underbrace{C(6,1)}_{\text{Choose 1 man}} = 56 \cdot 6 = 336 \text{ teams}$$

The number of teams with six women and no men is

$$\underbrace{C(8,6)}_{\text{Choose 6 women}} \cdot \underbrace{C(6,0)}_{\text{Choose no men}} = 28 \cdot 1 = 28 \text{ teams}$$

To count the number of team in the event "team has more than 4 women", add the number of teams in these two team compositions,

Teams with 5 women and 1 man ↘ ; Teams with 6 women and no men ↙

$$n(\text{Teams with more than 4 women}) = 336 + 28 = 364 \text{ teams}$$

The probability of randomly selecting a team with more than four women is

$$P(\text{Team has more than 4 women}) = \frac{n(\text{Teams with more than 4 women})}{n(\text{Teams with 6 members})}$$
$$= \frac{364}{3003}$$
$$\approx 0.121$$