

Question 3: How do you find the probability of winning a lottery?

Many people buy their weekly lottery ticket with the hope of getting rich quick. A dollar or two a week does not seem like a high price to pay for a chance at winning millions. The Powerball lottery played in 42 states recently had a jackpot of almost \$600 million dollars that was split by two winners.

In the Mega Millions lottery game, you pick five numbers from the 1 through 56 and another number (from the Mega Ball) from 1 through 46. If all of the numbers match the numbers drawn by lottery officials, the player wins the jackpot. To find out the number of different equally likely outcomes there are to randomly drawing the numbers, think of the selection of the numbers as two choices. First, select the first five numbers from the 56 numbers. Since the order in which the numbers are drawn do not make a difference, this can be done in

$$C(56,5) = \frac{56!}{(56-5)!5!} = 3,819,816 \text{ ways}$$

Second, choose the last number from the 46 numbers. This can be done in 46 ways. The total number of different ways the numbers may be picked is the product of the numbers of ways each choice may be done,

$$\frac{C(56,5)}{\text{Choose five numbers}} \cdot \frac{46}{\text{Choose last number}} = 3,819,816 \cdot 46 = 175,711,536 \text{ ways}$$

Each of these ways is equally likely. If we consider the experiment to be picking lottery numbers and the event A to be matching all of the numbers, the probability of matching all six numbers is

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{175,711,536} \approx 0.000000006$$

In the example below, we examine another set of numbers that results in a much lower payoff in the Mega Millions Lottery.

Example 3 Lottery

In the Mega Million Lottery, a player wins \$250,000,000 if they match five numbers, but not the Mega Ball number. Find the probability of winning \$250,000 in the lottery.

Solution We have already calculated the number of ways to select the numbers for the Mega Millions Lottery, 175,711,536 ways. To find the number of ways to match five numbers and not the Mega Ball number, break the selection of balls into two choices.

First, choose the numbers that match the numbers selected by lottery officials. Since there is only one set of numbers that match, there is only one way to match the first five numbers. Second, choose the Mega Ball number so that it does not match. There are 46 Mega Ball numbers so 45 of the numbers will not match. Apply the Multiplication Principle to these choices to give

$$\frac{1}{\text{Match first 5 numbers}} \cdot \frac{45}{\text{Do not match Mega Ball}} = 45$$

The probability of matching five numbers, but not the Mega Ball is

$$\begin{aligned} P(\text{Match 5 numbers}) &= \frac{n(\text{Match 5 numbers})}{n(\text{Select 6 numbers})} \\ &= \frac{45}{175,711,536} \\ &\approx 0.0000003 \end{aligned}$$

Compared to the likelihood of winning the jackpot, it is 45 times more likely to match the first five numbers. However, this likelihood is still very small. As a player, this is more of an “unlikelihood” than a likelihood!

