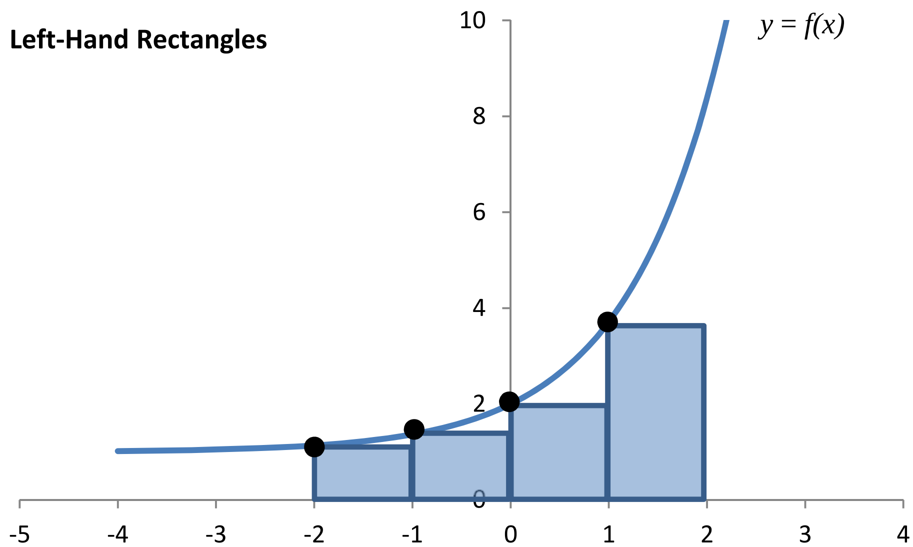
Approximate the area under the graph of $f(x) = e^x + 1$ and above the x-axis from $x = -2$ to $x = 2$ using four rectangles.

The area of the region may be approximated using rectangles that touch in different places along the top of the rectangle. For instance, the rectangles may touch the function on the left side as shown in the graph below.

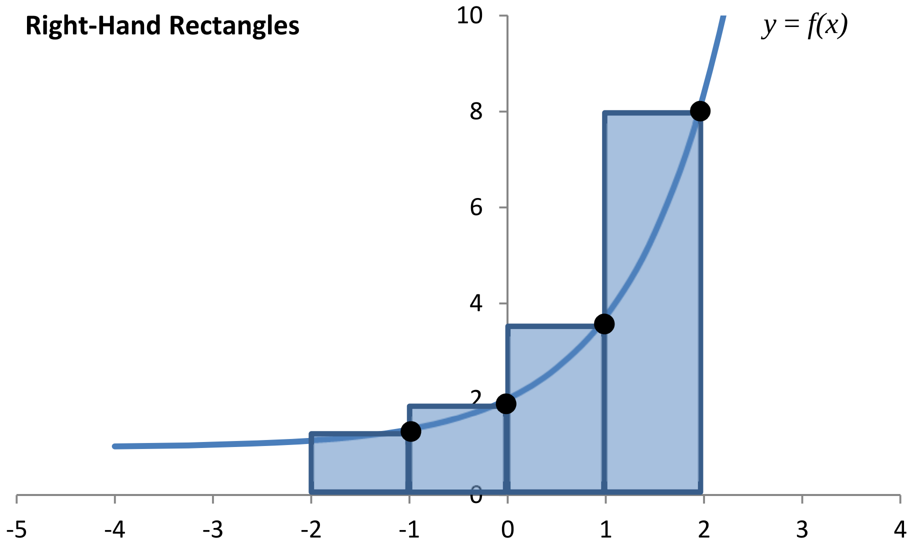


In this case, the heights of the rectangles are given by the heights of the function at $= -2, -1, 0, 1$. Each rectangle has a width of 1 unit so the approximate area is

$$f(-2) \cdot 1 + f(-1) \cdot 1 + f(0) \cdot 1 + f(1) \cdot 1 \approx 1.14 \cdot 1 + 1.37 \cdot 1 + 2 \cdot 1 + 3.72 \cdot 1$$
$$\approx 8.23$$

Since the top of each rectangle is below the curve, this sum is an underestimate of the exact area under the function.

We can carry out the same sum, but with the heights given by the right side of the rectangle. In this case we use the heights of the function at $x = -1, 0, 1, 2$.

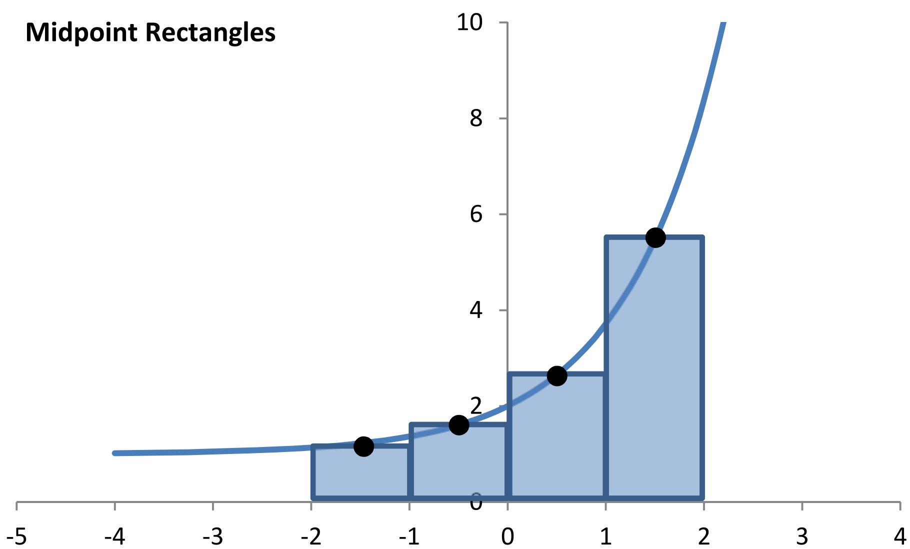


The area of the four rectangles is

$$f(-1)\cdot 1 + f(0)\cdot 1 + f(1)\cdot 1 + f(2)\cdot 1 \approx 1.37\cdot 1 + 2\cdot 1 + 3.72\cdot 1 + 8.39\cdot 1$$
$$\approx 15.48$$

The top of each rectangle is above the function so this sum is an overestimate of the exact area.

In a midpoint estimate, the height of each rectangle is determined by where the middle of the rectangle passes through the function.



This means the height are determined by substituting $x = -1.5, -0.5, 0.5, 1.5$ into the function. The sum of the areas is

$$f(-1.5)\cdot 1 + f(-0.5)\cdot 1 + f(0.5)\cdot 1 + f(1.5)\cdot 1 \approx 1.22\cdot 1 + 1.61\cdot 1 + 2.65\cdot 1 + 5.48\cdot 1$$
$$\approx 10.96$$