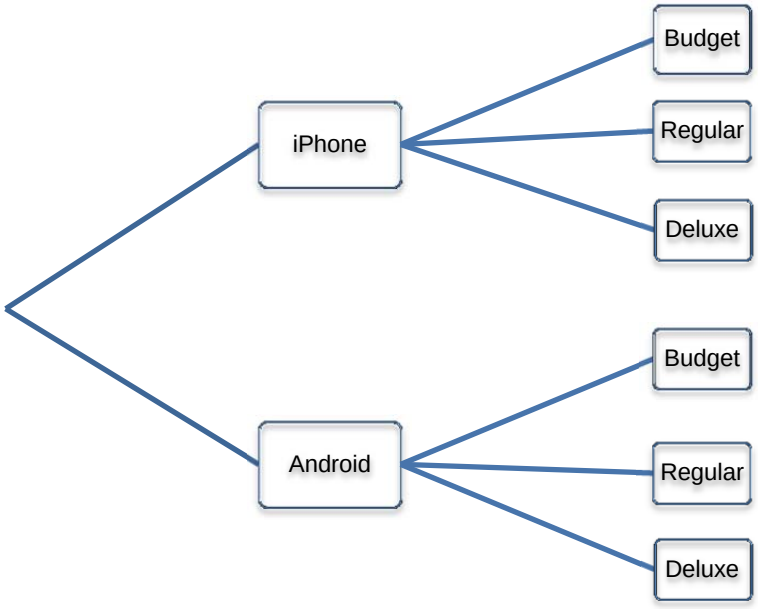
Question 1: How do you count choices using the multiplication principle?

A small cellular provider gives its customers 2 choices of phones to use. They may use an iPhone or a phone that uses the Android operating system. In addition, the company offers three different calling plans: Budget plan, Regular plan, and the Deluxe plan. How many different choices of phone and calling plan does a customer have?

To answer this question, we can use a decision tree and list out all of the choices a customer may make.



A decision tree show the different choice a customer makes when choosing a phone and plan. If we move left to right through the tree, we can list out each of the possibilities:

iPhone with Budget plan	iPhone with Regular plan	iPhone with Deluxe plan
Android with Budget plan	Android with Regular plan	Android with Deluxe plan

By listing out each of the possibilities, we see that there are six possible phone/plan choices. The decision tree helps us to list out these possibilities. However, if we only

need to know how many choices, we can multiply the number of choices for phones and plans..

$$\underline{\quad 2 \quad} \cdot \underline{\quad 3 \quad} = 6$$

This strategy is useful for determining the total number of choices even when there are a larger number of choices.

Multiplication Principle

Suppose we wish to know the number of ways to make n choices where there are

d_1 ways to make choice 1

d_2 ways to make choice 2

$\vdots$

d_n ways to make choice n

Then the total number of ways to make all of the choices is

$$d_1 \cdot d_2 \cdot \ldots \cdot d_n$$

Example 1 Multiplication Principle

An online custom bicycle seller wishes to count the total number of different types of bicycles that are available through its website. The seller offers 4 different frame styles, 8 different fender colors, 10

different tire colors, 8 different wheel colors, 6 different pedal colors, and 12 different accessory colors. How many different bicycles can a customer order?

Solution Each choice the customer must make leads to a different factor in the multiplication principle.

$$\underset{\text{frame style}}{4} \cdot \underset{\text{fender color}}{8} \cdot \underset{\text{tire color}}{10} \cdot \underset{\text{wheel color}}{8} \cdot \underset{\text{pedal color}}{6} \cdot \underset{\text{accessory color}}{12} = 184{,}320$$

There are 184,320 different bicycles that can be ordered.

Example 2 Multiplication Principle

As the number of cars on the road has increased, so has the number of license plates. The format of the license plate determines how many different license plates there are. For each of the formats below, find the number of different license plates that are available.

a. Three numbers

Solution We use the multiplication principle and choose each number. There are ten choices for numbers 0 through 9 giving

$$\underset{\text{first number}}{10} \cdot \underset{\text{second number}}{10} \cdot \underset{\text{third number}}{10} = 1000$$

b. Three letters followed by three numbers

Solution In this type of license plate, we have six choices to make. For each of the first three choices, there are 26 letters to choose from. For the last three choices, there are 10 numbers to choose from. This gives leads to the total number of license plates,

$$\underbrace{26}_{\text{first letter}} \cdot \underbrace{26}_{\text{second letter}} \cdot \underbrace{26}_{\text{third letter}} \cdot \underbrace{10}_{\text{first number}} \cdot \underbrace{10}_{\text{second number}} \cdot \underbrace{10}_{\text{third number}} = 17{,}576{,}000$$

c. Six characters where each character may be a letter or number

Solution Since the character can be a letter or a number, there are 36 choices for each character. This gives a total number of license plates,

$$\underbrace{36}_{\text{first choice}} \cdot \underbrace{36}_{\text{second choice}} \cdot \underbrace{36}_{\text{third choice}} \cdot \underbrace{36}_{\text{fourth choice}} \cdot \underbrace{36}_{\text{fifth choice}} \cdot \underbrace{36}_{\text{sixth choice}} = 2{,}176{,}782{,}336$$