Question 2: What is factorial notation?

Certain patterns occur often when applying the multiplication principle. As we saw in Example 2, the factors that result from choices are often the same. In this case, we can use exponents to abbreviate the product:

$$10 \cdot 10 \cdot 10 = 10^3$$
$$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^3 \cdot 10^3$$
$$36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 = 36^5$$

You may see the factors written with exponents instead of factors so it is important to recognize that they are the same.

Another pattern that results from the multiplication principle can be written using factorial notation. Suppose a production line requires six workers to carry out six different jobs. Each worker can only do one job at a time. Once a worker is selected for a job, the other jobs must be carried out by the remaining workers. To find the number of ways we can assign workers to jobs, calculate the product

$$\underset{\substack{\text{first}\\\text{job}}}{\underline{6}} \cdot \underset{\substack{\text{second}\\\text{job}}}{\underline{5}} \cdot \underset{\substack{\text{third}\\\text{job}}}{\underline{4}} \cdot \underset{\substack{\text{fourth}\\\text{job}}}{\underline{3}} \cdot \underset{\substack{\text{fifth}\\\text{job}}}{\underline{2}} \cdot \underset{\substack{\text{sixth}\\\text{job}}}{\underline{1}} = 720$$

The number of ways to make each choice drops by one in each factor since each worker can only do one job. In effect, we can’t choose the same worker twice. This is often indicated by saying that we want to assign workers without repetition.

This type of product occurs so often that it is assigned its own symbol.

Factorial Notation

For any positive integer n,

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

The value of $0!$ is defined to be 1.

When we read an expression with factorial notation, a symbol like $n!$ is read "n factorial".

Example 3 Use Factorial Notation

Compute the value of each expression involving factorial notation.

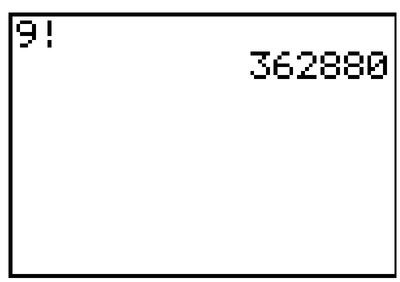
a. $6!$

Solution Use the formula above to get

$$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 720$$

b. $9!$

Solution It is tedious to multiply the factors out for larger numbers. Instead, use a calculator's factorial command to find the product. On a TI graphing calculator, start by typing 9. Then press



MATH ▶ ▶ ▶ 4 ENTER. Choosing 4 inserts the factorial symbol ! from the PRB menu. The value is displayed on the screen.

c. $\dfrac{100!}{98!}$

Solution It is not practical to multiply all of the factors in the numerator and denominator. In addition, each of the factors in the fraction may not be calculated individually. If we try to do this the calculator will return an overflow error. Instead, write down some of the factor to see if any patterns emerge:

$$\frac{100!}{98!} = \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}{98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}$$

Every factor in the denominator is also in the numerator. These factors may be reduced to give

$$\begin{aligned}\frac{100!}{98!} &= \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}{98 \cdot 97 \cdots 3 \cdot 2 \cdot 1} \\ &= \frac{100 \cdot 99 \cdot \cancel{98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}}{\cancel{98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}} \\ &= 100 \cdot 99 \\ &= 9900\end{aligned}$$