

Question 3: What is a permutation?

The term “permutation” refers to different arrangements in objects. We have already seen one example of a permutation in Question 2. When we allocated six workers among six jobs on a production line, we were counting the number of ways that we could arrange the workers among the jobs. This was a permutation of 6 workers taken 6 at a time. For permutations, we assume that the objects are arranged without repetition. In the context of the production line, this means that once a worker is given a job that worker cannot be assigned to another job.

Let's modify this application slightly. How many ways can we assign six workers to four jobs without repetition? As before, we need to choose workers for each job.

$$\frac{6 \cdot 5 \cdot 4 \cdot 3}{\begin{array}{cccc} \text{first} & \text{second} & \text{third} & \text{fourth} \\ \text{job} & \text{job} & \text{job} & \text{job} \end{array}} = 360$$

This is the same pattern that resulted in factorial notation, but we are missing the last two factors. However, we can still write this product in terms of factorial notation.

$$6 \cdot 5 \cdot 4 \cdot 3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{6!}{2!} = \frac{6!}{(6-4)!}$$

The number in the numerator indicates the number of objects we are selecting from. The number in the denominator is the difference between the number of objects we are selecting from and the number we are selecting. This relationship leads to a general rule for permutations.

Permutations

An arrangement of n objects taken r at a time without repetition is called a permutation. The number of these arrangements is symbolized $P(n, r)$ and found with

$$P(n, r) = \frac{n!}{(n-r)!}$$

where $r \leq n$. The symbol $P(n, r)$ is read “the permutation of n objects taken r at a time”.

It can be confusing to use the letter P to indicate permutations and probability. For this reason, some textbooks will write $P_{n,r}$ or P_r^n instead of $P(n, r)$. In practice, this is less of a concern since the values in parentheses are numbers for permutations and events represented by capital letters in probability. In this text, we'll stick with writing permutations as $P(n, r)$. If there is any possible confusion, we'll make sure to distinguish the permutations from the probabilities.

Recall our initial production line example. In that example we wanted to find the number of ways to select six workers. In terms of permutations, this would be the permutation of six workers taken six at a time or $P(6, 6)$. This would be calculated

$$P(6, 6) = \frac{6!}{(6-6)!} = \frac{6!}{0!} = \frac{6!}{1} = 6!$$

Remember that $0!$ is defined to be equal to 1

For permutations, order is important. Rearranging the same workers among the jobs leads to a different arrangement since each job is different. If order did not make a difference, then there would only be one way to allocate the six workers to six jobs. This

might be the case if all of the workers were doing the exact same type of job. In this case, moving the workers to another job would give the same arrangement. To apply permutations, look for wording that indicates that order is important and that objects are selected without repetition.

Example 4 Picking Stocks

Suppose you wish to purchase two different stocks from Agilent Technologies Inc (A), Citigroup (C), and Ford (F). You will invest \$10,000 in the first stock and \$5000 in the second stock.

- a. How many ways are there to choose the two stocks?

Solution Different amounts are being invested in each stock, This means that investing \$10,000 in Citigroup and \$5000 in Ford is different from investing \$10,000 in Ford and \$5000 in Citigroup. Therefore, order is important since it means that we invest different amounts. We also need to invest in two different stocks so we choose the stocks without repetition. We need to find the number of permutations of the three stocks taken two at a time,

$$P(3,2) = \frac{3!}{(3-2)!} = 6$$

- b. List all of the permutations of choosing 2 stocks from the list of three stocks.

Solution To make the list easier to show, we'll write each possibility as a two-letter "word" using the stock symbols. The first letter in the "word" represents the stock we'll invest \$10,000 in, The second letter represents the stock we'll invest \$5000 in. The permutations are

AC, AF, CA, CF, FA, FC



Example 5 Ranking Job Candidates

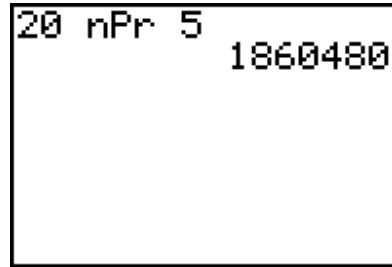
A hiring committee is asked to rank the top five candidates for a teaching position from a pool of 20 candidates. How many different arrangements are there?

Solution Since we want to rank the candidates, different arrangements must be counted differently. Additionally, once a candidate is ranked we cannot rank them in a different place. For these reasons, we need to calculate the permutation of 20 candidates taken 5 at a time,

$$P(20,5) = \frac{20!}{(20-5)!} = \frac{20!}{15!} = 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 = 1,860,480$$

We can use a TI graphing calculator to find the same number by typing

`2 [0] [MATH] [▼] [▼] [▼] 2 [5] [ENTER]`. This pastes the permutation command nPr from the MATH / PRB menu between 20 and 5.



20 nPr 5
1860480

The calculator indicates that the number of objects must be typed first by placing the n in front of the P . Since the r is placed after P , the number that are taken at a time is typed after the permutation command nPr .

