Corporations where a computer is essential to day-to-day operations, such as banks, often have a second backup computer in case the main computer fails. Suppose there is a .003 chance that the main computer will fail in a given time period, and a .005 chance that the backup computer will fail while the main computer is being repaired. Assuming these failures represent independent events, find the fraction of the time that the corporation can assume it will have computer service.

Define the events and probabilities: Let A = "the main computer fails" and B = "the backup computer fails". Then the information given in the problem can be written as

$$P(A) = .003, \quad P(B \mid A) = .005$$

To have computer service, the main computer must not fail OR the backup computer must not fail. This can be represented by the event $A' \cup B'$. So we need to find $P(A' \cup B')$ to find the fraction of the time the corporation can assume it will have computer service.

Use the union rule for probability: The union rule can be used to write

$$P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$$

To use this rule, we need to find each of the probabilities on the right hand side.

Find $P(A')$ and $P(B')$: If $P(A) = .003$ and we know the sum of all probabilities is 1, then

$$P(A') = 1 - .003 = .997$$

Since A and B are independent, $P(B \mid A) = P(B)$. In effect, the event A has nothing to do with the event B. Now we can write

$$P(B') = 1 - .005 = .995$$

Find $P(A' \cap B')$: Here we'll use the definition of conditional probability to write

$$P(A' \cap B') = P(B' \mid A')P(A')$$

If A and B are independent, so are the compliments of A and B so we know that $P(B' \mid A') = P(B')$. Using this leads us to

$$P(A' \cap B') = P(B')P(A') = .995 \cdot .997$$

Finish using the union rule for probability: Now we finish using the union rule to get

$$P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$$
$$= .997 + .995 - .997 \cdot .995$$
$$= .999985$$

This means that the corporation will have computer service 99.9985% of the time. This is impressive given that the main computer operates 99.7% of the time and the backup operates 99.5% of the time. It also explains why corporations put backup systems in place.