

A restaurant owner orders a replacement set of knives, fork, and spoons. The box arrives containing 40 utensils and weighing 141.3 ounces (ignoring the weight of the box). A knife, fork, and spoon weigh 3.9 ounces, 3.6 ounces, and 3.0 ounces, respectively.

How many knives, forks and spoons are in the box?

Since we are being asked to find the number of knives, forks, and spoons, let's make the following designations:

K: number of knives

F: number of forks

S: number of spoons

The first thing to notice is that you are given a total number of utensils (40) and a total weight for the utensils (141.3 ounces). These are the prime candidates for writing out the equations.

Starting with

$$\text{Total number of utensils} = 40$$

It is fairly obvious that

$$K + F + S = 40$$

Starting with

$$\text{Total weight of utensils} = 141.3$$

We can deduce that the individual weights are

$$\text{Weight of knives} = 3.9 K$$

$$\text{Weight of forks} = 3.6 F$$

$$\text{Weight of spoons} = 3.0 S$$

So

$$3.9 K + 3.6 F + 3.0 S = 141.3$$

Now what? Your experience in these sections probably tells you that you need another equation in the three unknowns to be able to solve for K, F, and S. This is true if there is a unique solution to this problem. But this problem actually has many possible solutions (ie. There are many ways to have 40 utensils that weigh 141.3 ounces).

So let's simply solve the problem as is with Gauss-Jordan elimination:

$$K + F + S = 40$$

$$3.9K + 3.6F + 3.0S = 141.3$$

Start by converting this to an augmented matrix: $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 3.9 & 3.6 & 3.0 & 141.3 \end{array} \right]$

To put into row echelon form, let's multiply the first row by -3.9 and add it to the second row. Put the result in row 2:

$$-3.9R_1 + R_2 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & -0.3 & -0.9 & -14.7 \end{array} \right]$$

To make the first nonzero number in the second row a 1, multiply the entire row by 1/-0.3 and replace the second row with that result:

$$-\frac{1}{0.3}R_2 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 0 & 1 & 3 & 49 \end{array} \right]$$

This is now in row echelon form. It is almost as if there was a third row in the matrix, but it is all zeros. Converting this back to a system of equations, we get

$$K + F + S = 40$$

$$F + 3S = 49$$

To get our solutions, solve the second equation for F to yield $F = 49 - 3S$. Putting this back into the first equation gives $K + (49 - 3S) + S = 40$ or $K + 49 - 2S = 40$. Solving this for K gives $K = 2S - 9$. The solution to the system is

$$K = 2S - 9$$

$$F = 49 - 3S$$

And S can be any value that makes sense for the problem. For instance, we certainly know that S (the number of spoons) should be non-negative integers like $S = 0, 1, 2, \dots$. If we make up a table, we can make some interesting observations:

Spoons	S	0	1	2	5	6	7	14	15	16	17
Knives	$2S - 9$	-9	-7	-5	1	3	5	19	21	23	25
Forks	$49 - 3S$	49	46	43	34	31	28	7	4	1	-2

We need the numbers of each type of utensil to be positive...to do that we'll require the number of spoons to be $S = 5, 6, 7, \dots, 16$. So even though the system has an infinite number of solutions, only certain ones make sense.