Section 3.1 Matrix Addition and Subtraction

Question 1 – How is information organized in a matrix?

Question 2 – How do you multiply a matrix by a scalar?

Question 3 - How do you add matrices?

Question 4 - How do you subtract matrices?

Question 1 – How is information organized in a matrix?

Key Terms

Size Transpose

Summary

A matrix is a table of numbers organized into rows and columns. Matrices are symbolized with capital letters. An example would be the matrix A below:

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 4 & 5 \end{bmatrix}$$

The entries in a matrix are placed within brackets. The size of a matrix is described by the number of rows and columns in the matrix. The matrix A above is a 2 × 3 matrix (read 2 by 3) since it has 2 rows and 3 columns. When quoting the size of a matrix, the number of rows are listed first and the number of columns is listed second.

Two matrices are equal if their corresponding entries (entries in the same row and column) are equal.

We can refer to the entries using lower case letters and subscripts. The notation a_{mn} is the entry in the matrix A in the mth row and nth column. In the matrix above, $a_{21} = 6$ is the entry in the second row and third column.

We can organize information in matrices by labeling the rows and column in a matrix. For instance, The Mundo Candy Company makes three types of chocolate candy: Cheery Cherry (CC), Mucho Mocha (MM), and Almond Delight (AD). Each kilogram of Cheery Cherry requires .5 kg of sugar and .2 kg of chocolate, each kilogram of Mucho Mocha requires .4 kg of sugar and .3 kg of chocolate; and each kilogram of Almond Delight requires .3 kg of sugar and .3 kg of chocolate.

Write a matrix which corresponds the type of candy with the amount of sugar and chocolate in the candy.

Start by deciding what size matrix to use. Since we are trying to match up 3 types of candy with 2 ingredients, we can use a 3×2 or a 2×3. For a 3×2 matrix, we label the rows with the type of candy and the columns with the ingredients:

$$\begin{array}{c} \\ \text{CC} \\ \text{MM} \\ \text{AD} \end{array} \begin{array}{c} \begin{array}{cc} \text{sugar} & \text{choc} \end{array} \\ \begin{bmatrix} .5 & .2 \\ .4 & .3 \\ .3 & .3 \end{bmatrix} \end{array}$$

For a 2×3, label the row with the ingredients and the columns with the types of candy,

$$\begin{array}{c} \\ \text{sugar} \\ \text{choco} \end{array} \begin{array}{c} \begin{array}{ccc} \text{CC} & \text{MM} & \text{AD} \end{array} \\ \begin{bmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{bmatrix} \end{array}$$

Either matrix corresponds the ingredients with the types of candy.

These two matrices are transposes of each other. This means that if we change the rows in one matrix to columns, we get the other matrix.

$$\begin{bmatrix} .5 & .2 \\ .4 & .3 \\ .3 & .3 \end{bmatrix} \xrightarrow[\text{SWITCH ROWS AND COLUMNS}]{\text{TRANSPOSE}} \begin{bmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{bmatrix}$$

<u>Notes</u>

Guided Example	Practice
Suppose you are given the matrix $$A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ -1 & 5 \end{bmatrix}$$ a. Find the size of the matrix. **Solution** The matrix has three rows and two columns, so it is a 3 × 2 matrix. b. Find the value of a_{12} . **Solution** The entry a_{12} is the entry in the first row and second column so $a_{12} = -2$. c. Find the value of a_{32} . **Solution** The entry a_{32} is the entry in the third row and second column so $a_{12} = 5$.	1. Suppose you are given the matrix $$B = \begin{bmatrix} -1 & 3 & -2 \\ 2 & 1 & -3 \\ 0 & 4 & 5 \end{bmatrix}$$ a. Find the size of the matrix. b. Find the value of b_{23} . c. Find the value of b_{31} .

Guided Example	Practice
The Mundo Candy Company makes three types of chocolate candy: Cheery Cherry, Mucho Mocha, and Almond Delight. The company produces its products in San Diego, Mexico City, and Managua using two main ingredients: chocolate and sugar. The cost of 1 kg of sugar is \$4 in San Diego, \$2 in Mexico City, and \$1 in Managua. The cost of 1 kg of chocolate is \$3 in San Diego, \$5 in Mexico City, and \$7 in Managua. a. Find a 2 × 3 matrix that relates the cost of ingredients to the city. **Solution** Label the rows with the ingredients and the columns with the cities to give $\begin{array}{c c} & \begin{array}{ccc} SD & \text{MC} & M \end{array} \\ \begin{array}{c} \text{sugar} \\ \text{choco} \end{array} & \begin{bmatrix} 4 & 2 & 1 \\ 3 & 4 & 7 \end{bmatrix} \end{array}$ b. Find a 3 × 2 matrix that relates the cost of ingredients to the city. **Solution** Label the rows with the cities and the columns with the ingredients to give $\begin{array}{c c} & \begin{array}{cc} \text{sugar} & \text{choco} \end{array} \\ \begin{array}{c} \text{SD} \\ \text{MC} \\ \text{M} \end{array} & \begin{bmatrix} 4 & 3 \\ 2 & 4 \\ 1 & 7 \end{bmatrix} \end{array}$	2. The A-Plus auto parts store has two outlets, one in Phoenix and one in Tucson. Among other things, it sells wiper blades, air fresheners, and floor mats. During January, the Phoenix outlet sells 20 wiper blades, 10 air fresheners, and 8 floor mats. The Tucson outlet sells 15 wiper blades, 12 air fresheners, and 4 floor mats. During February, the Phoenix outlet sells 23 wiper blades, 8 air fresheners, and 4 floor mats. The Tucson outlet sells 12 wiper blades, 12 air fresheners, and 5 floor mats. a. Write a 2 x 3 matrix that reflects the January sales. Label the rows and columns of your matrix. b. Write a 2 x 3 matrix that February sales. Label the rows and columns of your matrix.

Question 2 – How do you multiply a matrix by a scalar?

Key Terms

Scalar

Summary

Multiplying a matrix by a scalar means multiplying the matrix by a number. To carry this multiplication out, multiply each entry in the matrix by the number.

Notes

Guided Example

Guided Example	Practice
For the matrix $$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 3 & 4 \end{bmatrix}$$ Compute $3A$. **Solution** Multiply each entry in the matrix to give $$\begin{aligned} 3A &= 3\begin{bmatrix} 1 & 0 & -3 \\ 2 & 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3\cdot 1 & 3\cdot 0 & 3(-3) \\ 3\cdot 2 & 3\cdot 3 & 3\cdot 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 & -9 \\ 6 & 9 & 12 \end{bmatrix} \end{aligned}$$	1. For the matrix $$B = \begin{bmatrix} 2 & 7 \\ 5 & -2 \end{bmatrix}$$ Compute $2B$.

Question 3 – How do you add matrices?

Key Terms

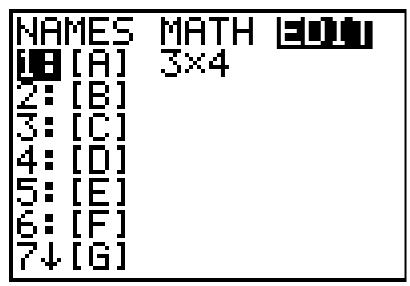
Summary

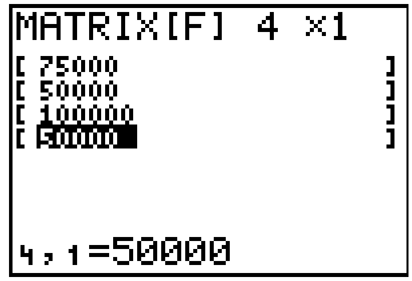
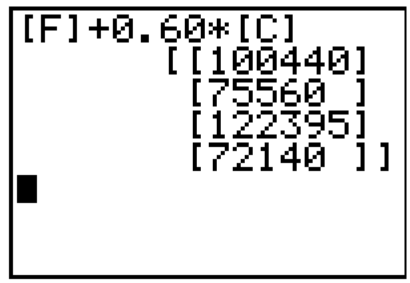
To be able to add two matrices, they must have the same size. If they do have the same size, they are added by adding the corresponding entries in the matrices.

Matrices can also be added, multiplied by a scalar ro subtracted on a TI graphing calculator. Suppose

$$C = \begin{bmatrix} 42400 \\ 42600 \\ 37325 \\ 36900 \end{bmatrix} \text{ and } F = \begin{bmatrix} 75000 \\ 50000 \\ 100000 \\ 50000 \end{bmatrix}$$

Find $F + 0.60C$ by following the steps below on a TI graphing calculator.

Enter the Matrix 1. Press ψ □ to access the MATRIX menu. 2. Use ~ to go to EDIT. 3. Press ℜ or move the cursor to 3: [C] and press ⊆. Note that if you used this matrix name before, it will have a dimension next to it.	
4. Enter the dimension of matrix C as 4 x 1. 5. Enter the values into the matrix as shown. Press ⊆ after each entry. Note that the position is given at the bottom of the screen as 4,1=36900 etc. 6. Press ψ ζ to QUIT and return to the home screen.	

7. Repeat steps 1 through 5 to enter the matrix called [F].	
Perform the Matrix Arithmetic on the Home Screen 8. Press ψ ζ to QUIT and return to the Home screen. 9. You must use MATRIX NAMES to enter the names of the matrices. Press ψ □ to access NAMES. 10. Select 6: [F] and press ⊆. 11. Use the operation key ℘ to add. Follow this command with ⊇⊄÷⊇↓ 12. Repeat steps 9 and 10 the process to select matrix 3:[C] and press ⊆. 13. To carry out the operations, press ⊆.	

<u>Notes</u>

Guided Example	Practice
Suppose $A=\begin{bmatrix}0 & -2\\ 3 & 1\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & 1\\ 2 & -2\\ 0 & -2\end{bmatrix}$. a. Compute *A* + *B*. **Solution** Both matrices have size 3 × 2 so they may be added. Add the corresponding entries to give $$A+B=\begin{bmatrix}0+3 & -2+1\\ 3+2 & 1+(-2)\\ -1+0 & 1+(-2)\end{bmatrix} =\begin{bmatrix}3 & -1\\ 5 & -1\\ -1 & -1\end{bmatrix}$$ b. Compute 2*A* + 5*B* **Solution** To make it easier to carry out the matrix addition, first multiply the scalars and matrices. Then add the corresponding entries: $$2A+5B=\begin{bmatrix}0 & -4\\ 6 & 2\\ -2 & 8\end{bmatrix}+\begin{bmatrix}15 & 5\\ 10 & -10\\ 0 & -10\end{bmatrix} =\begin{bmatrix}0+15 & -4+5\\ 6+10 & 2+(-10)\\ -2+0 & 8+(-10)\end{bmatrix} =\begin{bmatrix}15 & 1\\ 16 & -8\\ -2 & -2\end{bmatrix}$$	1. Suppose $C=\begin{bmatrix}7 & -2\\ 1 & -1\end{bmatrix}$ and $D=\begin{bmatrix}2 & -1\\ 1 & -3\end{bmatrix}$. a. Compute $C+D$. b. Compute $3C+4D$

Question 4 – How do you subtract matrices?

Key Terms

Summary

To be able to subtract two matrices, they must have the same size. If they do have the same size, they are subtracted by subtracting the corresponding entries in the matrices.

Notes

Guided Example	Practice
Suppose $A=\begin{bmatrix}0 & -2\\ 3 & 1\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}3 & 1\\ 2 & -2\\ 0 & -2\end{bmatrix}$. a. Compute *A* - *B*. **Solution** The sizes of the matrices match so we can subtract corresponding entries, $A-B=\begin{bmatrix}0-3 & -2-1\\ 3-2 & 1-(-2)\\ -1-0 & 4-(-2)\end{bmatrix}$ $=\begin{bmatrix}-3 & -3\\ 1 & 3\\ -1 & 6\end{bmatrix}$ b. Compute $\frac{1}{2}A-B$ **Solution** Multiply A by the scalar and then subtract corresponding entries, $\frac{1}{2}A-B=\begin{bmatrix}0 & -1\\ \frac{3}{2} & \frac{1}{2}\\ -\frac{1}{2} & 2\end{bmatrix}-\begin{bmatrix}3 & 1\\ 2 & -2\\ 0 & -2\end{bmatrix}$ $=\begin{bmatrix}0-3 & -1-1\\ \frac{3}{2}-2 & \frac{1}{2}-(-2)\\ -\frac{1}{2}-0 & 2-(-2)\end{bmatrix}$ $=\begin{bmatrix}-3 & -2\\ -\frac{1}{2} & \frac{5}{2}\\ -\frac{1}{2} & 4\end{bmatrix}$	1. Suppose $C=\begin{bmatrix}7 & -2\\ 1 & -1\end{bmatrix}$ and $D=\begin{bmatrix}2 & -1\\ 1 & -3\end{bmatrix}$. a. Compute *C* - *D*. b. Compute $2C-0.5D$

Section 3.2 Matrix Multiplication

Question 1 – How do you multiply two matrices?

Question 2 – How do you interpret the entries in a product of two matrices?

Question 1 – How do you multiply two matrices?

<u>Key Terms</u>

Matrix product

<u>Summary</u>

Suppose we have a 1 x k matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix}$$

and a k x 1 matrix,

$$B = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix}$$

In each matrix, the dots help to indicate the arbitrary number of rows or columns in each matrix. Although this number k is arbitrary, the number of columns in A must match the number of rows in B. Otherwise it is not possible to carry out the multiplication process.

To find the product these matrices, we must multiply the entries in the row matrix by the entries in the column matrix and add the resulting products:

$$AB = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{n1} \end{bmatrix}$$

$$= a_{11}\,b_{11} + a_{12}\,b_{21} + \cdots + a_{1n}\,b_{n1}$$

Notice that each product comes from corresponding columns and rows. In other words, the first product is formed from the first column in the first matrix and the first row in the second matrix, the second product is formed from the second column in the first matrix and the second column in the second matrix, and so on.

Let's try the following product:

$$\begin{bmatrix} 3 & 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -9 \\ -2 \\ 1 \end{bmatrix}$$

To help identify the factors in the products, let's color code each corresponding factor and carry out the sum:

$$\begin{bmatrix} 3 & 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -9 \\ -2 \\ 1 \end{bmatrix} = 3(2) + 1(-9) + (-2)(-2) + 4(1) = 5$$

The key to carrying out the process is to correspond the factors in each product correctly.

How to Multiply Two Matrices

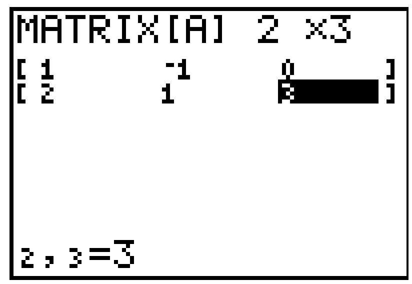
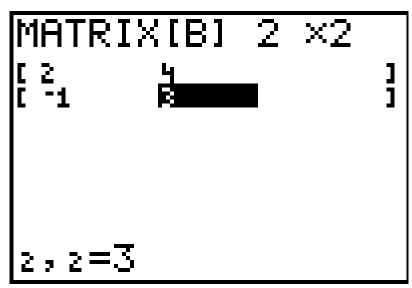
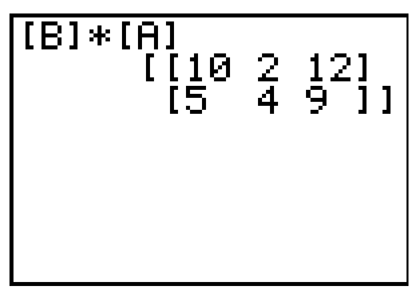
1. Make sure the number of columns in the first matrix matches the number of rows in the second column. If they do not match, the product is not possible.
2. The size of the products is the number of rows in the first matrix by the number of columns in the second matrix. The product of m x k matrix and a k x n matrix is an m x n matrix. Form a matrix of the proper size with blank spaces for each entry.
3. For each entry in the product, form the corresponding factors and sums. The entry in the ith row and jth column of the product is found by corresponding and multiplying the ith row in the first matrix with the jth column in the second matrix.

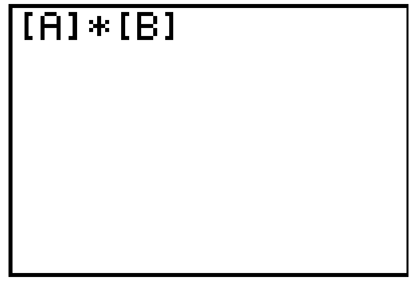
If an m × k matrix is multiplied by a k × n matrix, the product will be a m × n.

$$(m \times k) \quad (k \times n) = m \times n$$

As noted earlier, the number od columns in the first matrix must match the number of rows in the second matrix.

Let’s look at how we can carry out matrix multiplication with technology. For the matrices $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$, find the product BA using a TI graphing calculator.

Enter the matrices 1. Press ψ □ to access the MATRIX menu. 2. Use the right arrow to go to EDIT. 3. Move the cursor to 1: [A] and press ⊆. Note that if you used this matrix name before, it will have a dimension next to it. 4. Enter the dimension of matrix A as 2 x 3. Enter the values into the matrix as shown. Note that the position is given at the bottom of the screen as 2,3=3 etc.	
5. Repeat the process to enter matrix B, i.e. Press ψ □ to return to the MATRIX menu etc. 6. Press ψ ζ to QUIT and return to the Home screen.	
Perform the matrix multiplication on the home screen 7. Use ψ□ to access NAMES. Enter the names of the matrices. You may use the multiplication key to multiply the	

matrices as shown, but it not necessary. 8. Press ⊆ to see the product matrix.	
9. Note that you get a dimension error when you try to do [A] [B]. This is because the number of columns in [A] does not match the number of rows in [B].	 

<u>Notes</u>

Guided Example	Practice
Suppose $A=\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}10\\ 5\end{bmatrix}$. a. Find AB. **Solution** Matrix A is a 2 × 2 matrix and B is a 2 × 1 matrix. The product will be a 2 × 1 matrix. $AB=\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}\begin{bmatrix}10\\ 5\end{bmatrix} =\begin{bmatrix}1\cdot 10+3\cdot 5\\ -1\cdot 10+2\cdot 5\end{bmatrix} =\begin{bmatrix}25\\ 0\end{bmatrix}$ b. Find BA. **Solution** For the product BA to make sense, the number of columns in B, 1, must match the number of rows in A, 2. Since these do not match, it is not possible to compute the product.	1. Suppose $A=\begin{bmatrix}3\\ 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0\\ -2 & 3\end{bmatrix}$. a. Find AB. b. Find BA.

Guided Example	Practice
Suppose $A=\begin{bmatrix}0 & 5 & -1\\ 2 & 1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}4 & -1\\ 2 & 10\end{bmatrix}$. a. Find *AB*. **Solution** For the product *AB* to make sense, the number of columns in *A*, 3, must match the number of rows in *B*, 2. Since these do not match, it is not possible to compute the product. b. Find *BA*. **Solution** Since the number of columns in *B* matches the number of rows in *A*, it is possible to compute the product. In this problem, we are multiplying a 2 × 2 by a 2 × 3 which results in a 2 × 3. $BA=\begin{bmatrix}4 & -1\\ 2 & 10\end{bmatrix}\begin{bmatrix}0 & 5 & -1\\ 2 & 1 & 4\end{bmatrix}$ $=\begin{bmatrix}4\cdot 0+(-1)2 & 4\cdot 5+(-1)(1) & 4(-1)+(-1)4\\ 2\cdot 0+10\cdot 2 & 2\cdot 5+10\cdot 1 & 2(-1)+10\cdot 4\end{bmatrix}$ $=\begin{bmatrix}-2 & 19 & -8\\ 20 & 20 & 38\end{bmatrix}$	Suppose $A=\begin{bmatrix}1 & 3\\ 2 & 0\\ -1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}6 & 3\\ -1 & 2\end{bmatrix}$. a. Find *AB*. b. Find *BA*.

Question 2 – How do you interpret the entries in a product of two matrices?

Key Terms

Summary

Let's look at an example of how to apply matrix multiplication.

> The Mundo Candy Company makes three types of chocolate candy: Cheery Cherry, Mucho Mocha, and Almond Delight. The company produces its products in San Diego, Mexico City, and Managua using two main ingredients: chocolate and sugar.
>
> Each kilogram of Cheery Cherry requires .5 kg of sugar and .2 kg of chocolate, each kilogram of Mucho Mocha requires .4 kg of sugar and .3 kg of chocolate; and each kilogram of Almond Delight requires .3 kg of sugar and .3 kg of chocolate. The cost of 1 kg of sugar is \$4 in San Diego, \$2 in Mexico City, and \$1 in Managua. The cost of 1 kg of chocolate is \$3 in San Diego, \$5 in Mexico City, and \$7 in Managua.
>
> Put the information above in a matrix in such a way that when you multiply the matrices, you get a matrix representing the ingredient cost of producing each type of candy in each city.

Start by putting the information in a matrix. There are two ingredients and three types of candy so we need either a 2 x 3 or 3 x 2. Either will be fine as long as we label the rows and columns. I choose to use a 2 x 3:

$$\begin{array}{r c} & \begin{array}{ccc} \text{CC} & \text{MM} & \text{AD} \end{array} \\ \begin{array}{r} \text{sugar} \\ \text{choco} \end{array} & \begin{bmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{bmatrix} \end{array}$$

Because the product has to correspond to candy type and cities, the product must be a 3 x 3 matrix. To get this from the 2 x 3 above, we'll need to multiply a 3 x 2 times the 2 x 3. Based on the information above, the rows must correspond to cities and the columns to ingredients:

$$\begin{array}{r c} & \begin{array}{cc} \text{sugar} & \text{choco} \end{array} \\ \begin{array}{r} \text{SD} \\ \text{MC} \\ \text{M} \end{array} & \begin{bmatrix} 4 & 3 \\ 2 & 5 \\ 1 & 7 \end{bmatrix} \end{array}$$

Now let's carry out the multiplication:

$$\begin{bmatrix} 4 & 3 \\ 2 & 5 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{bmatrix} = \begin{bmatrix} 2.6 & 2.5 & 2.1 \\ 2 & 2.3 & 2.1 \\ 1.9 & 2.3 & 2.4 \end{bmatrix}$$

To get the entry in the second row, first column of the product we need to multiply the second row in the first matrix by the first column in the second matrix and add the results:

$$2(.5)+5(.2)=2$$

Other entries are calculated similarly. Since we are multiplying amounts of ingredients times cost per amount, the product is a total cost. How should we label the product?

$$(\text{city}\times\text{ingredient})(\text{ingredient}\times\text{candy})=\text{city}\times\text{candy}$$

so

$$\begin{array}{c c} & \begin{array}{ccc}\text{CC} & \text{MM} & \text{AD}\end{array} \\ \begin{array}{c}\text{SD}\\ \text{MC}\\ \text{M}\end{array} & \begin{bmatrix} 2.6 & 2.5 & 2.1 \\ 2 & 2.3 & 2.1 \\ 1.9 & 2.5 & 2.4 \end{bmatrix}\end{array}$$

The cost of Cheery Cherry is Mexico City would be $2.

<u>Notes</u>

Guided Example

A political candidate plans to use three methods of advertising: newspapers, radio, and cable TV. The cost per ad (in thousands of dollars) for each type of media is given by matrix A.

$$\begin{array}{cc} & \text{Unit Cost} \\ A = & \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{news} \\ \text{radio} \\ \text{cable} \end{array}$$

Matrix B shows the number of ads per month in these three media that are targeted to single people, to married males aged 35 to 55, and to married females over 65 years of age.

$$\begin{array}{cc} & \begin{array}{ccc} \text{news} & \text{radio} & \text{cable} \end{array} \\ B = & \begin{bmatrix} 10 & 15 & 20 \\ 20 & 5 & 10 \\ 5 & 10 & 25 \end{bmatrix} \end{array} \begin{array}{c} \\ \text{single} \\ \text{married men} \\ \text{married female} \end{array}$$

Find the matrix that gives the cost of ads for each target group.

Solution The matrix A is a 3 × 1 matrix (media × unit cost) and matrix B is a 3 × 3 matrix (targeted group × media). If we multiply B times A, the number of columns in B (3 representing media) matches the number of rows in A (also representing media).

$$(\text{targeted group} \times \text{media})(\text{media} \times \text{unit cost})$$

The result will be a 3 × 1 matrix representing (targeted group × cost).

$$\begin{aligned} BA &= \begin{bmatrix} 10 & 15 & 20 \\ 20 & 5 & 10 \\ 5 & 10 & 25 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} 10\cdot 3+15\cdot 1+20\cdot 5 \\ 20\cdot 3+5\cdot 1+10\cdot 5 \\ 5\cdot 3+10\cdot 1+25\cdot 5 \end{bmatrix} \\ &= \begin{bmatrix} 145 \\ 115 \\ 150 \end{bmatrix} \end{aligned}$$

Labeling the matrix, we get

$$
\begin{array}{c c l}
 & \text{Cost} & \\
BA = & \begin{bmatrix} 145 \\ 115 \\ 150 \end{bmatrix} & \begin{array}{l} \text{single} \\ \text{married men} \\ \text{married female} \end{array}
\end{array}
$$

<u>Practice</u>

1. Men and women in a church choir wear choir robes in the sizes shown in matrix A.

$$
\begin{array}{c c l}
 & \begin{array}{cccc} S & M & L & XL \end{array} & \\
A = & \begin{bmatrix} 8 & 15 & 5 & 0 \\ 1 & 10 & 10 & 5 \end{bmatrix} & \begin{array}{l} \text{Women} \\ \text{Men} \end{array}
\end{array}
$$

Matrix B contains the prices (in dollars) of new robes and hoods according to size.

$$
\begin{array}{c c l}
 & \begin{array}{cc} \text{robes} & \text{hoods} \end{array} & \\
B = & \begin{bmatrix} 55 & 25 \\ 65 & 25 \\ 75 & 35 \\ 95 & 35 \end{bmatrix} & \begin{array}{l} S \\ M \\ L \\ XL \end{array}
\end{array}
$$

Find the matrix that gives the total cost of robes and hoods for men and women.

Section 3.3 Matrix Inverses

Question 1 – What is a matrix inverse?

Question 2 – How do you find a matrix inverse?

Question 1 – What is a matrix inverse?

Key Terms

Square matrix	Identity matrix
Inverse of a matrix	Invertible

Summary

A square matrix is a matrix in which the number of rows is equal to the number of columns.

An identity matrix is a square matrix with ones along the diagonal entries and zeros elsewhere. The letter I is used to denote an identity matrix. If the context of the problem is not clear enough to establish the size of the identity matrix, a subscript is used to establish the size. For instance, I_2 would be a 2×2 identity matrix and I_3 would be a 3×3 identity matrix.

Two square matrices A and B are called inverses if and only if their product is the identity matrix,

$$AB = BA = I$$

The matrix B is called the inverse of A and is written A^{-1}. A matrix that has an inverse is called an invertible matrix.

Notes

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<tr><th><u>Guided Example</u></th><th><u>Practice</u></th></tr>
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Are A and B inverses of each other?

$$A=\begin{bmatrix}3&5\\1&2\end{bmatrix}\quad B=\begin{bmatrix}2&-5\\-1&3\end{bmatrix}$$

Solution Matrices that are inverses of each other yield the identity matrix when they are multiplied. Start by multiplying *AB*:

$$\begin{aligned}AB&=\begin{bmatrix}3&5\\1&2\end{bmatrix}\begin{bmatrix}2&-5\\-1&3\end{bmatrix}\\&=\begin{bmatrix}3\cdot2+5(-1)&3(-5)+5\cdot3\\1\cdot2+2(-1)&1(-5)+2\cdot3\end{bmatrix}\\&=\begin{bmatrix}1&0\\0&1\end{bmatrix}\end{aligned}$$

Now check *BA*:

$$\begin{aligned}BA&=\begin{bmatrix}2&-5\\-1&3\end{bmatrix}\begin{bmatrix}3&5\\1&2\end{bmatrix}\\&=\begin{bmatrix}2\cdot3+(-5)(1)&2\cdot5+(-5)2\\-1\cdot3+3\cdot1&-1\cdot5+3\cdot2\end{bmatrix}\\&=\begin{bmatrix}1&0\\0&1\end{bmatrix}\end{aligned}$$

Since *AB* = *BA* = *I*, the matrices are inverses.

</td><td>

1. Are A and B inverses of each other?

$$A=\begin{bmatrix}5&7\\3&4\end{bmatrix}\quad B=\begin{bmatrix}-4&7\\3&-5\end{bmatrix}$$

</td></tr>
</table>

Guided Example

If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$, are A and B inverses?

Solution Compute the products AB and BA to determine if they are equal to the identity matrix. In both cases, the product of two 3 x 3 matrices is another 3 x 3 matrix.

$$AB = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot 4 + 1(-1) + (-1)(2) & 1(-2) + 1\cdot 1 + (-1)(-1) & 1(-1) + 1\cdot 1 + (-1)(0) \\ 2\cdot 4 + 2(-1) + (-3)(2) & 2(-2) + 2\cdot 1 + (-3)(-1) & 2(-1) + 2\cdot 1 + (-3)(0) \\ (-1)(4) + 0(-1) + 2\cdot 2 & (-1)(-2) + 0\cdot 1 + 2(-1) & (-1)(-1) + 0\cdot 1 + 2\cdot 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4\cdot 1 + (-2)(2) + (-1)(-1) & 4\cdot 1 + (-2)(2) + (-1)(0) & 4(-1) + (-2)(-3) + (-1)(2) \\ (-1)(1) + 1\cdot 2 + 1(-1) & (-1)(1) + 1\cdot 2 + 1\cdot 0 & (-1)(-1) + 1(-3) + 1\cdot 2 \\ 2\cdot 1 + (-1)(2) + 0(-1) & 2\cdot 1 + (-1)(2) + 0\cdot 0 & 2(-1) + (-1)(-3) + 0\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Both products are equal to the 3 x 3 identity matrix. so

$$A^{-1} = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$$

Practice

2. If $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$, are A and B inverses?

Question 2 – How do you find a matrix inverse?

Key Terms

Summary

To find the inverse of a square matrix, the matrix is combined with an identity matrix of the same size in a single matrix. If the matrix is called *A*, we write this symbolically as $\left[A \mid I\right]$. If the matrix A is a 2 x 2 matrix, combining it with a 2 x 2 identity matrix results in a 2 x 4 matrix.

The inverse matrix is found by using row operations to transform the matrix so that the identity matrix is on the left side of the combined matrix. The right side of the matrix is the inverse of the matrix *A*. Symbolically, we must use row operations to yield $\left[I \mid A^{-1}\right]$.

How to Find the Inverse of a Matrix

To compute A^{-1} for an *n* x *n* matrix *A*,

1. Place the matrix *A* alongside the identity matrix I_n to form a new matrix $\left[A \,\vdots\, I_n\right]$
2. Use row operations to place a one in the first row, first column of the matrix.
3. Use row operations to place zeros in the rest of the first column.
4. Continue using row operations to place a one in each column in the row that matches the column number. Once the one is in place in a column, use row operations to make the rest of the entries in that column equal to zero.
5. When the left-hand side of the matrix is equal to I_n, the right hand side of the matrix is the inverse of *A* or A^{-1}.
6. If any of the rows on the left-hand side of the matrix consists entirely of zeros, then the matrix *A* does not have an inverse. When a matrix does not have an inverse, we say it is not invertible.

A graphing calculator may be used to find the inverse of a matrix. Suppose we start with the matrix

$$A = \begin{bmatrix} -2 & 1 & 2 \\ 1 & 0 & -1 \\ 4 & -2 & -3 \end{bmatrix}.$$

To calculate the inverse of A, follow the instructions below.

<table>
<tr>
<td>
Enter the matrix

Press ψ □ to access the MATRIX menu.

Move the cursor to 1: [A] and press ⊆.

Enter the dimension of matrix A as 3 x 3. Enter the values into the matrix as shown. Note that the position is given at the bottom of the screen as 3,3= -3 etc.

Press ψ ζ to QUIT and return to the Home screen.
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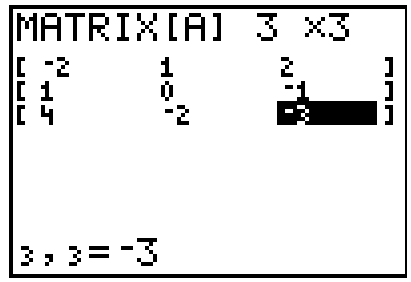

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<tr>
<td>
Find the matrix inverse on the home screen

Press ψ□ to enter the name of the matrix. Press 1 or ⊆ to paste [A] to the home screen.

Press the □ key to find the inverse of matrix A.

Press ⊆ to see A^{-1}.
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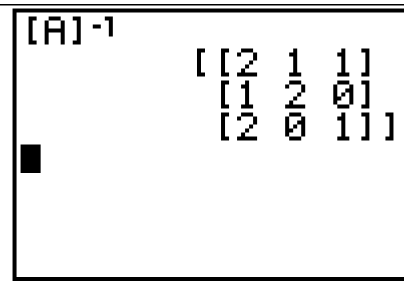

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<tr>
<td>
Show that $AA^{-1} = I$

On the home screen use ψ⊆ to paste the name of the matrix A.

On the same line, enter matrix A again, then press the □ key.

Press ⊆ to see to see the product matrix, the identity matrix for a 3x3 matrix.
</td>
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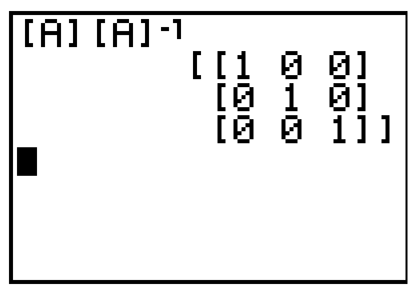

</td>
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</table>

<u>Notes</u>

<u>Guided Example</u>

Find the inverse of the matrix $A=\begin{bmatrix}-1 & -2\\ 3 & 4\end{bmatrix}$

Solution Start by augmenting the matrix A with a 2 × 2 identity matrix: $\left[\begin{array}{cc:cc}-1 & -2 & 1 & 0\\ 3 & 4 & 0 & 1\end{array}\right]$.

Now use row operations to transform the matrix to $\left[I \mid A^{-1}\right]$.

$$-1R_1 \to R_1 \qquad \left[\begin{array}{cc:cc}1 & 2 & -1 & 0\\ 3 & 4 & 0 & 1\end{array}\right]$$

$$-3R_1 + R_2 \to R_2 \qquad \left[\begin{array}{cc:cc}1 & 2 & -1 & 0\\ 0 & -2 & 3 & 1\end{array}\right]$$

$$-\tfrac{1}{2}R_2 \to R_2 \qquad \left[\begin{array}{cc:cc}1 & 2 & -1 & 0\\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2}\end{array}\right]$$

$$-2R_2 + R_1 \to R_1 \qquad \left[\begin{array}{cc:cc}1 & 0 & 2 & 1\\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2}\end{array}\right]$$

The right-hand side of the matrix is A^{-1},

$$A^{-1}=\begin{bmatrix}2 & 1\\ -\frac{3}{2} & -\frac{1}{2}\end{bmatrix}$$

<u>Practice</u>

1. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$

Guided Example

Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

Solution Form the initial matrix $\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$. Now carry out row operation to but a 3 × 3 identity matrix on the left side of this matrix.

$$\begin{array}{c} -2R_1 + R_2 \to R_2 \\ -1R_1 + R_3 \to R_3 \end{array} \qquad \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 3 & -2 & 1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$-1R_2 \to R_2 \qquad \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$$\begin{array}{c} -1R_2 + R_1 \to R_1 \\ R_2 + R_3 \to R_3 \end{array} \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & -1 & 1 \end{array}\right]$$

$$-1R_3 \to R_3 \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -1 & 1 & 0 \\ 0 & 1 & -3 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

$$\begin{array}{c} 3R_3 + R_2 \to R_2 \\ -2R_3 + R_1 \to R_1 \end{array} \qquad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & -1 & 2 & -3 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right]$$

So the inverse matrix is

$$A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{bmatrix}$$

Practice

2. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$.

Section 3.4 Solving Matrix Equations with Inverses

Question 1 – How do you write a system of equations as a matrix equation?

Question 2 – How do you solve a matrix equation using the matrix inverse?

Question 1 – How do you write a system of equations as a matrix equation?

Key Terms

Matrix equation	Variable matrix
Constant matrix	Coefficient matrix

Summary

A system of linear equations can be written as a matrix equation. First, make sure the terms with variables are on the left side of the equation and any constants are on the right side of the equations. Second, write the variable terms in the same order in each equation. Third, define three matrices:

- Variable matrix – a column matrix containing the variables in the system
- Constant matrix – a column matrix containing the constants from the right side of the equations
- Coefficient matrix – a matrix containing the coefficients of the variable from the left side of the equations

Typically, the variable matrix is named X, the coefficient matrix is called A, and the constant matrix is called B.

Suppose we have the system

$$\begin{aligned} 2x+3y&=7 \\ x+y&=3 \end{aligned}$$

Notice that the variable terms are on the left side with x's listed first and y's listed second. The constants are all written on the right side. Now let's define the matrices X, A, and B:

$$X=\begin{bmatrix} x \\ y \end{bmatrix}, \quad A=\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, \quad B=\begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

With these matrices defined, the original system is equivalent to the matrix equation $AX = B$. At first this may not be clear. However, let's carry out the matrix multiplication AX:

$$AX=\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 2x+3y \\ x+y \end{bmatrix}$$

The product yields the left side of the original system of equations. Setting this equal to B means

$$\begin{bmatrix} 2x+3y \\ x+y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

For these matrices to be equal, the corresponding entries must be equal:

$$\begin{array}{ccc} AX & = & B \\ \begin{bmatrix} 2x+3y \\ x+y \end{bmatrix} & = & \begin{bmatrix} 7 \\ 3 \end{bmatrix} \end{array} \text{ is equivalent to } \begin{aligned} 2x+3y&=7 \\ x+y&=3 \end{aligned}$$

<u>Notes</u>

Guided Example	Practice
Suppose you are given the system $$\begin{aligned}2x - y &= 9\\ x - y &= 4\end{aligned}$$ Write this system in matrix form $AX = B$ by defining the matrices A, X, and B. **Solution** Define the matrices $$X = \begin{bmatrix} x \\ y \end{bmatrix}$$ $$A = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}$$ $$B = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$ Checking the equation $AX = B$ allows to see that these matrices result in an equivalent matrix equation, $$\begin{aligned} AX &= B \\ \begin{bmatrix} 2x - y \\ x - y \end{bmatrix} &= \begin{bmatrix} 9 \\ 4 \end{bmatrix} \end{aligned} \text{ is equivalent to } \begin{aligned} 2x - y &= 9 \\ x - y &= 4 \end{aligned}$$	1. Suppose you are given the system $$\begin{aligned}x - 3y &= 2\\ 2x - y &= 9\end{aligned}$$ Write this system in matrix form $AX = B$ by defining the matrices A, X, and B.

Guided Example	Practice
Suppose you are given the system $$\begin{aligned} x-2y+z&=0\\ 2x+y-z&=2\\ x+y+z&=3 \end{aligned}$$ Write this system in matrix form $AX = B$ by defining the matrices A, X, and B. Solution Define the matrices $$X=\begin{bmatrix} x\\ y\\ z \end{bmatrix}$$ $$A=\begin{bmatrix} 1 & -2 & 1\\ 2 & 1 & -1\\ 1 & 1 & 1 \end{bmatrix}$$ $$B=\begin{bmatrix} 0\\ 2\\ 3 \end{bmatrix}$$ Checking the equation $AX = B$ allows to see that these matrices result in an equivalent matrix equation, $$\begin{array}{ccc} AX & = & B\\ \begin{bmatrix} x-2y+z\\ 2x+y-z\\ x+y+z \end{bmatrix} & = & \begin{bmatrix} 0\\ 2\\ 3 \end{bmatrix} \end{array} \text{ is equivalent to } \begin{aligned} x-2y+z&=0\\ 2x+y-z&=2\\ x+y+z&=3 \end{aligned}$$	2. Suppose you are given the system $$\begin{aligned} x+z&=4\\ y-z&=-1\\ x+y+z&=6 \end{aligned}$$ Write this system in matrix form $AX = B$ by defining the matrices A, X, and B.

Question 2 – How do you solve a matrix equation using the matrix inverse?

Key Terms

Summary

Solving a matrix equation $AX = \mathbf{B}$ means that we need to solve for the variable matrix X. On the surface, you might simply think we need to divide by the matrix A on both sides. However, matrix division is not a defined operation. Instead, multiply both sides of the equation by the inverse of the coefficient matrix A^{-1} :

$$A^{-1}AX = A^{-1}B$$

Since $A^{-1}A = I$, we can write this equation as

$$IX = A^{-1}B$$

The identity matrix I times any matrix is that matrix, so $IX = X$. The solution to the matrix equation $AX = B$ is

$$X = A^{-1}B$$

As long as we are able to compute the inverse A^{-1}, we can obtain the solution to the matrix equation by multiplying it by B.

We can do this on a graphing calculator as shown below. Suppose we want to solve the system

$$\begin{aligned} 5x + 7y + 12z &= 134{,}000 \\ 2x + 3y + 5z &= 56{,}000 \\ x + y + z &= 14{,}000 \end{aligned}$$

using the inverse matrix of the coefficient matrix

$$A = \begin{bmatrix} 5 & 7 & 12 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

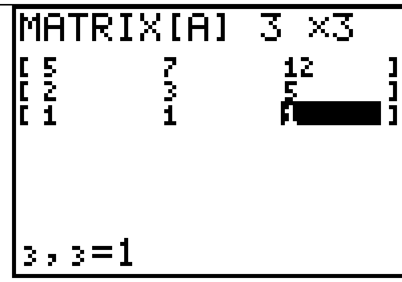
and the constant matrix

$$B = \begin{bmatrix} 134{,}000 \\ 56{,}000 \\ 14{,}000 \end{bmatrix}$$

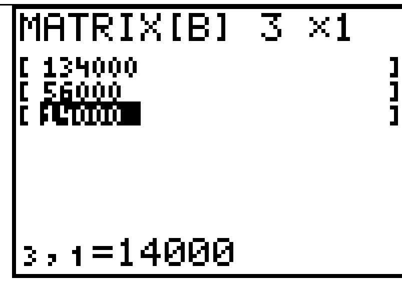
The instructions below show how to use a graphing calculator and $X = A^{-1}B$ to find the solution to the system of equations.

Enter the matrices A and B

1. Press ψ □ to edit the matrix.
2. Move to the EDIT tab and select 1: [A].
3. Enter matrix A.

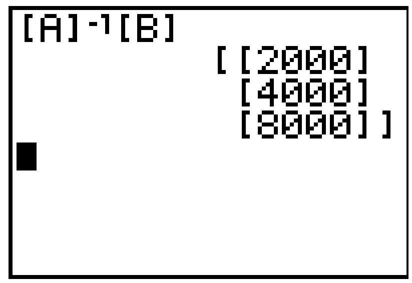


4. Repeat the process and enter matrix B.
5. Press ψ ζ to QUIT and return to the home screen.



Find the product matrix of A^{-1} and B on the home screen

6. Press ψ□ to enter the name of the matrix A.
7. Use the □ key to place the inverse sign after of matrix A as shown.
8. Press ψ□to enter the name of the matrix B. Note that you do not need to see the inverse matrix of A.
9. Press ⊆ to see the product.
10. The entries in the product correspond to the entries is the variable matrix, $x = 2000$, $y = 4000$, and $z = 8000$.



<u>Notes</u>

Guided Example	Practice
Solve the system below using the inverse of the coefficient matrix. $$\begin{aligned} 2x - y &= 9 \\ x - y &= 4 \end{aligned}$$ **Solution** This system is equivalent to the matrix equation $AX = B$ with $$X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad A = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$ The inverse of the coefficient matrix is $$A^{-1} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}$$ Multiply this matrix by B to find the variable matrix $$\begin{aligned} X &= \quad A^{-1} \quad B \\ &= \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 1 \end{bmatrix} \end{aligned}$$ The solution to the system is $x = 5$ and $y = 1$.	1. Solve the system below using the inverse of the coefficient matrix. $$\begin{aligned} x - 3y &= 2 \\ 2x - y &= 9 \end{aligned}$$

Guided Example	Practice

Solve the system below using the inverse of the coefficient matrix.

$$\begin{aligned} x_1 + x_2 - x_3 &= 6 \\ 2x_1 + x_2 + x_3 &= 17 \\ x_1 + x_3 &= 8 \end{aligned}$$

Solution This system is equivalent to the matrix equation $AX = B$ with

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 17 \\ 8 \end{bmatrix}$$

The inverse of the coefficient matrix is

$$A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{bmatrix}$$

Multiply this matrix by B to find the variable matrix

$$\begin{aligned} X &= \quad A^{-1} \quad B \\ &= \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 17 \\ 8 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 4 \\ 3 \end{bmatrix} \end{aligned}$$

The solution to the system is $x_1 = 5$, $x_2 = 4$, and $x_3 = 3$.

2. Solve the system below using the inverse of the coefficient matrix.

$$\begin{aligned} x + y + z &= 60 \\ 2x + y + z &= 70 \\ x + y + 2z &= 90 \end{aligned}$$

Guided Example

An electronics company produces transistors, resistors, and computer chips. Each transistor requires 3 units of copper, 1 unit of zinc, and 2 units of glass. Each resistor requires 3, 2, and 1 units of the three materials, and each computer chip requires 2, 1, and 2 units of these materials, respectively. How many of each product can be made with 5500 units of copper, 2500 units of zinc, and 3500 units of glass? Solve this exercise by using the inverse of the coefficient matrix to solve a system of equations.

Solution Start by defining the variables:

T: number of transistors
R: number of resistors
C: number of computer chips

We know that a total of 5500 units of copper is available. Since a transistor uses 3 units of copper, a resistor uses 3 units of copper, and a computer chip uses 2 units of copper, the relationship for how much copper is needed is

$$3T+3R+2C=5500$$

We can follow similar reasoning for the total amount of zinc, 2500 units. This results in the equation $T+2R+C=2500$. The relationship for the total amount of glass is $2T+R+2C=3500$.

To find the number of transistors, resistors, and computer chips, we need to solve the system of equations,

$$\begin{aligned} 3T+3R+2C&=5500 \\ T+2R+C&=2500 \\ 2T+R+2C&=3500 \end{aligned}$$

Define the matrices

$$X=\begin{bmatrix} T \\ R \\ C \end{bmatrix}, \quad A=\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \quad B=\begin{bmatrix} 5500 \\ 2500 \\ 3500 \end{bmatrix}$$

The inverse of the coefficient matrix is

$$A^{-1}=\begin{bmatrix} 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 & 1 \end{bmatrix}$$

The solution is

$$X=A^{-1}B=\begin{bmatrix} 1 & -\frac{4}{3} & -\frac{1}{3} \\ 0 & \frac{2}{3} & -\frac{1}{3} \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 5500 \\ 2500 \\ 3500 \end{bmatrix}=\begin{bmatrix} 1000 \\ 500 \\ 500 \end{bmatrix}$$

Or 1000 transistors, 500 resistors, and 500 computer chips.

Practice

3. The Riddler Rent-A-Truck company plans to spend $7 million on 200 new vehicles. Each commercial van will cost $35,000, each small truck $30,000, and each large truck $50,000. They need twice as many vans as small trucks. How many of each vehicle can they buy? Solve this problem by using the inverse of the coefficient matrix to solve a system of equations.

Chapter 3 Solutions

Section 3.1

Question 1 1) a. 3 × 3, b. -3, c. 0

2) a. $\begin{array}{c|ccc} & \text{wiper} & \text{air} & \text{floor} \\ \hline \text{Phx} & 10 & 10 & 8 \\ \text{Tuc} & 15 & 12 & 4 \end{array}$, b. $\begin{array}{c|ccc} & \text{wiper} & \text{air} & \text{floor} \\ \hline \text{Phx} & 23 & 8 & 4 \\ \text{Tuc} & 12 & 12 & 5 \end{array}$

Question 2 1) $\begin{bmatrix} 4 & 14 \\ 10 & -4 \end{bmatrix}$

Question 3 1) a. $\begin{bmatrix} 9 & -3 \\ 2 & -4 \end{bmatrix}$, b. $\begin{bmatrix} 29 & -10 \\ 7 & -15 \end{bmatrix}$

Question 4 1) a. $\begin{bmatrix} 5 & -1 \\ 0 & 2 \end{bmatrix}$, b. $\begin{bmatrix} 13 & -3.5 \\ 1.5 & -0.5 \end{bmatrix}$

Section 3.2

Question 1 1) a. Not possible, b. $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ 2) a. $\begin{bmatrix} 3 & 9 \\ 12 & 6 \\ -11 & 7 \end{bmatrix}$, b. Not possible

Question 2 1) $\begin{array}{c|cc} & \text{Robes} & \text{Hoods} \\ \hline \text{Women} & 1550 & 750 \\ \text{Men} & 1930 & 800 \end{array}$

Section 3.3

Question 1 1) Yes, 2) Yes

Question 2 1) $A^{-1} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$, 2) $A^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 3 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

Section 3.4

Question 1 1) 2)

Question 2 1) 2) 3)