## Section 6.1 Representing Data

Question 1 - What is a frequency distribution?
Question 2 - How do you make a histogram?
Question 3 - What is a bar chart?

Question 1 - What is a frequency distribution?

## Key Terms

Relative frequency Frequency distribution

## Summary

Suppose a dataset contains $k$ different values $x_{1}, x_{2}, \ldots, x_{k}$. Each data value $x_{i}$ occurs $f_{i}$ times in the data set. A table with the data values $x_{i}$ in the first column and the corresponding frequencies $f_{i}$ in the second column is called a frequency distribution.

The relative frequency of the data value $x_{i}$ is

$$
\text { Relative Frequency }=\frac{f_{i}}{n}
$$

where $n$ is the total number of measurements in the dataset. A table with the data values $x_{i}$ in the first column and the corresponding relative frequencies $\frac{f_{i}}{n}$ in the second column is called a relative frequency distribution.
In cases where there are many data values, the data values may be grouped into classes. The width of a class is

$$
\text { width }=\frac{\text { largest data value }- \text { smallest data value }}{\text { number of classes }}
$$

However, often the width is modified slightly so that all of the classes fall on "nice" numbers or so every data value falls into a class. When data are grouped, the first column in the frequency distribution contains the different classes and the second column is how frequently a data value falls in the class. For a relative frequency distribution, the first column contains the different classes and the second column is the relative frequency how often a data value falls in a class.

## Notes

## Guided Example

A bank measures the amount of time it takes 40 randomly selected customers to make a deposit. These times are recorded in the table below.

| 2 | 2 | 3 | 1 | 2 | 2 | 2 | 1 | 2 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 2 | 2 | 2 | 5 | 4 | 3 | 2 | 3 |
| 3 | 4 | 3 | 3 | 2 | 3 | 1 | 3 | 2 | 2 |
| 2 | 3 | 1 | 2 | 1 | 2 | 2 | 5 | 1 | 2 |

a. Construct a frequency distribution table.

Solution This table contains five distinct data values (1, 2, 3, 4, 5). These will form the first column of the frequency distribution table. To find the second column of the table containing the frequencies, count how many times each distinct data value occurs in the table of data.

| Data Value | Frequency |
| :---: | :---: |
| 1 | 7 |
| 2 | 20 |
| 3 | 9 |
| 4 | 2 |
| 5 | 2 |

b. Construct a relative frequency table.

Solution The number of values in the data table is 40 . To get the relative frequency distribution, divide each frequency in the frequency distribution table by 40.

| Data Value | Relative Frequency |
| :---: | :---: |
| 1 | 0.175 |
| 2 | 0.5 |
| 3 | 0.225 |
| 4 | 0.05 |
| 5 | 0.05 |

The relative frequency may also be written as percentages.
c. What percentage of customers made a deposit in two minutes or less?

Solution This question asks us to find the percentage of customers with the data value 1 or 2 . The relative frequency of those values is 0.175 and 0.5 . This means the percentage must be $17.5 \%+50 \%$ or $67.5 \%$ of customers.

## Practice

1. A manufacturing company measures the number of defective units its production facility produces in an hour. Over the course of several days, it records the data below.

| 0 | 3 | 3 | 3 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 1 | 1 |
| 0 | 1 | 1 | 2 | 2 |
| 0 | 0 | 1 | 1 | 1 |

a. Construct a frequency distribution table.
b. Construct a relative frequency table.
c. What percentage of hours had 1 or less defective units?

## Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

| $44 \%$ | $65 \%$ | $70 \%$ | $94 \%$ | $96 \%$ | $90 \%$ | $94 \%$ | $99 \%$ | $85 \%$ | $73 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $96 \%$ | $98 \%$ | $96 \%$ | $96 \%$ | $95 \%$ | $73 \%$ | $80 \%$ | $94 \%$ | $85 \%$ | $93 \%$ |
| $96 \%$ | $73 \%$ | $94 \%$ | $67 \%$ | $61 \%$ | $93 \%$ | $97 \%$ | $80 \%$ | $99 \%$ |  |
| $81 \%$ | $61 \%$ | $87 \%$ | $83 \%$ | $96 \%$ | $95 \%$ | $84 \%$ | $69 \%$ | $92 \%$ |  |
| $91 \%$ | $90 \%$ | $78 \%$ | $88 \%$ | $97 \%$ | $90 \%$ | $97 \%$ | $91 \%$ | $97 \%$ |  |

a. Construct a frequency distribution table with 6 classes.

Solution Group the data into 6 classes so that the width is approximately

$$
\text { width }=\frac{99-44}{6} \approx 9.2
$$

We will round this up to 10 for convenience so that the first class starts at 44 (and include 44) and extends to 54 (but does not include 54). In interval notation, we would write this as $[44,54$ ) where the bracket means include the endpoint and the parentheses means do not include the endpoint. Filling out the first column of the frequency table we get

| Class | Frequency |
| :---: | :---: |
| $[44,54)$ |  |
| $[54,64)$ |  |
| $[64,74)$ |  |
| $[74,84)$ |  |
| $[84,94)$ |  |
| $[94,104)$ |  |

This table consists of data values that are percent on an exam. Since percent on an exam cannot exceed $100 \%$, the last class is a bit problematic. We can solve this by creating the classes by starting at $[90,100)$ and working backwards. In this case we would end up with the classes below:

| Class | Frequency |
| :---: | :---: |
| $[40,50)$ |  |
| $[50,60)$ |  |
| $[60,70)$ |  |
| $[70,80)$ |  |
| $[80,90)$ |  |
| $[90,100)$ |  |

Both grouping allow all data points to fall into some class. Either grouping could be used to make the frequency distribution. We'll continue with the second one for this example and count how often each data in the table occurs in the corresponding class.

| Class | Frequency |
| :---: | :---: |
| $[40,50)$ | 1 |
| $[50,60)$ | 0 |
| $[60,70)$ | 5 |
| $[70,80)$ | 5 |
| $[80,90)$ | 9 |
| $[90,100)$ | 27 |

b. Construct a relative frequency table.

Solution There are 47 scores in the data table. Divide each frequency in the frequency table by 47 to give

| Class | Relative Frequency |
| :---: | :---: |
| $[40,50)$ | 0.021 |
| $[50,60)$ | 0 |
| $[60,70)$ | 0.106 |
| $[70,80)$ | 0.106 |
| $[80,90)$ | 0.191 |
| $[90,100)$ | 0.574 |

Each decimal has been rounded to three decimal places. If we were to use percentages, we would get the table below.

| Class | Relative Frequency |
| :---: | :---: |
| $[40,50)$ | $2.1 \%$ |
| $[50,60)$ | 0 |
| $[60,70)$ | $10.6 \%$ |
| $[70,80)$ | $10.6 \%$ |
| $[80,90)$ | $19.1 \%$ |
| $[90,100)$ | $57.4 \%$ |

Notice that because of rounding, the percentages do not add up to $100 \%$ exactly.
c. What percentage of scores are 70 or above?

Solution A score of 70 or above corresponds to the three highest classes. According to the relative frequency table, those classes have relative frequencies of approximately $10.6 \%, 19.1 \%$, and $57.4 \%$. The percentage of scores that are 70 or better is $10.6 \%+19.1 \%+57.4 \%$ or $87.1 \%$.

## Practice

2. Suppose the grades on an exam for a class are recorded in the table below.

| $35 \%$ | $38 \%$ | $63 \%$ | $92 \%$ |
| :---: | :---: | :---: | :---: |
| $76 \%$ | $55 \%$ | $65 \%$ | $77 \%$ |
| $85 \%$ | $98 \%$ | $71 \%$ | $75 \%$ |
| $88 \%$ | $89 \%$ | $91 \%$ | $75 \%$ |
| $71 \%$ | $79 \%$ | $88 \%$ | $85 \%$ |

a. Construct a frequency distribution table where the first class is $[25,40)$ and subsequent classes have the same width.
b. Construct a relative frequency table.
c. What percentage of scores are below $70 \%$ ?

Question 2 - How do you make a histogram?

## Key Terms

## Histogram

Summary
A histogram is a graph of the frequency or relative frequency distribution where classes are used to group the data. The horizontal axis is labeled with the borders of the classes from the distributions. The vertical axis is labeled as frequency or relative frequency with a scale that encompasses the values in the second column of the frequency or relative frequency table. Above each section of the horizontal axis, a bar is drawn vertically whose height corresponds to the frequency or relative frequency of data values that fall in the class. The width of the bar spans the class corresponding to the bar.

Notes

## Guided Example

Draw a histogram that corresponds to the frequency table below.

| Class | Frequency |
| :---: | :---: |
| $[40,50)$ | 1 |
| $[50,60)$ | 0 |
| $[60,70)$ | 5 |
| $[70,80)$ | 5 |
| $[80,90)$ | 9 |
| $[90,100)$ | 27 |

Solution Label the horizontal axis with the classes and the vertical axis with a scale that spans all the frequencies. Draw bars above each class with a height corresponding to the frequency.


## Practice

1. Suppose the grades on an exam for a class are recorded in the table below

| $35 \%$ | $38 \%$ | $63 \%$ | $92 \%$ |
| :---: | :---: | :---: | :---: |
| $76 \%$ | $55 \%$ | $65 \%$ | $77 \%$ |
| $85 \%$ | $98 \%$ | $71 \%$ | $75 \%$ |
| $88 \%$ | $89 \%$ | $91 \%$ | $75 \%$ |
| $71 \%$ | $79 \%$ | $88 \%$ | $85 \%$ |

Construct a histogram for the data where the first class is $[25,40)$ and subsequent classes have the same width.

Question 3 - What is a bar chart?

## Key Terms

Bar chart
Summary
A bar chart is a graph of the frequency or relative frequency distribution the data consists of a few distinct values or the data are qualitative. The horizontal axis is labeled with the data values. The vertical axis is labeled as frequency or relative frequency with a scale that encompasses the values in the second column of the frequency or relative frequency table. Above each data value on the horizontal axis, a bar is drawn vertically whose height corresponds to the frequency or relative frequency of data values that fall in the class. Since the data values are discrete, the bars are typically not drawn so that one bar does not touch the adjacent bar.

## Notes

## Guided Example

Draw a bar chart for the relative frequency table below.

| Data Value | Relative Frequency |
| :---: | :---: |
| 1 | 0.175 |
| 2 | 0.5 |
| 3 | 0.225 |
| 4 | 0.05 |
| 5 | 0.05 |

Solution Label each data value on the horizontal axis. Draw a bar above each data value so that the height matches the relative frequency.


## Practice

1. A manufacturing company measures the number of defective units its production facility produces in an hour. Over the course of several days, it records the data below.

| 0 | 3 | 3 | 3 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 1 | 1 |
| 0 | 1 | 1 | 2 | 2 |
| 0 | 0 | 1 | 1 | 1 |

Construct a bar chart for the data.

## Section 6.2 Measuring Central Tendency

Question 1 - What is the difference between a sample and a population?
Question 2 - What is the mean of a dataset?
Question 3 - What is the median of a dataset?
Question 4 - What is the mode of a dataset?

Question 1 - What is the difference between a sample and a population?

## Key Terms

Population Sample

## Summary

In statistics, we often want to ask questions about very large sets of data. For instance, perhaps we want to know how often a production line produces a defective unit. Over the course of a week, that production line may produce thousands of units. It is not feasible to test every one of the units to determine if it is defective or not.

Instead of looking at every item produced, we might pick a smaller group of units from the production line and determine how many are defective. As long as this group is picked randomly from the larger group of units produced, we can use the information about the smaller group to infer how many defective. The smaller group is called a sample which is selected from the larger group. The larger group is called the population.

Notes

Question 2 - What is the mean of a dataset?

## Key Terms

## Mean

## Summary

The mean of a set of data is calculated by adding up all of the data and then dividing by the total number of data. A population mean is calculated from a large set of data and is denoted by the letter $\mu$.

## Population Mean

Let $x_{i}$ denote the $i^{\text {th }}$ observation of a variable $x$ from a population with $N$ total observations. The population mean is

$$
\mu=\frac{x_{1}+x_{2}+\cdots+x_{N}}{N}
$$

A sample mean is denoted by the symbol $\bar{x}$ and is calculated the same way. However, in the case of the sample mean the data is a sample from a larger population.

## Sample Mean

Let $x_{i}$ denote the $i^{\text {th }}$ observation of a variable $x$ from a sample with $n$ total observations. The sample mean is

$$
\bar{x}=\frac{x_{1}+x_{2}+\cdots+x_{n}}{n}
$$

A graphing calculator may be used to calculate the value of the mean for a set of data.

1. Start by entering the data into a list on the calculator. Press STAT 1:Edit to enter a list.
2. Enter each data value under L 1 followed by ENTER.
3. Once each data value has been entered, press 2nd MODE to quit the list editor and return to the home screen.


| 4. From the home screen, press 2nd STAT to access the LIST |
| :--- |
| commands. |
| 5. Press |
| 6. Press to move the cursor to the MATH menu. |
| home screen. |

## Notes

## Guided Example

A bank measures the amount of time it takes 40 randomly selected customers to make a deposit. These times are recorded in the table below.

| 2 | 2 | 3 | 1 | 2 | 2 | 2 | 1 | 2 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | 1 | 2 | 2 | 2 | 5 | 4 | 3 | 2 | 3 |
| 3 | 4 | 3 | 3 | 2 | 3 | 1 | 3 | 2 | 2 |
| 2 | 3 | 1 | 2 | 1 | 2 | 2 | 5 | 1 | 2 |

Find the mean amount of time it takes a customer to make a deposit.
Solution Since this is a sample from a much larger set of data (all customers making deposits), we use the formula

$$
\bar{x}=\frac{x_{1}+x_{2}+\cdots+x_{n}}{n}
$$

to calculate the mean. The sum of the data is 92 and the total number of data values is 40 ,

$$
\bar{x}=\frac{92}{40}=2.3
$$

The mean time it takes a customer to make a deposit is 2.3 minutes.

## Practice

1. A manufacturing company measures the number of defective units its production facility produces in an hour. Over the course of several days, it records the data below.

| 0 | 3 | 3 | 3 | 0 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3 | 1 | 1 |
| 0 | 1 | 1 | 2 | 2 |
| 0 | 0 | 1 | 1 | 1 |

Find the mean number of defective units produced in an hour.

## Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

| Class | Frequency |
| :---: | :---: |
| $[40,50)$ | 1 |
| $[50,60)$ | 0 |
| $[60,70)$ | 5 |
| $[70,80)$ | 5 |
| $[80,90)$ | 9 |
| $[90,100)$ | 27 |

Estimate the mean score on the exam using the midpoint of each class to represent the class.
Solution In this example, the data are given in a frequency table. For instance, the third row of the table indicates that there are 5 data values in the interval $[60,70)$. We do not know exactly what the data values are so we will estimate all data values in $[60,70$ ) with the representative value 65 .

We could include five 65 's in the sum for the mean, however, it is easier to simply write $5 \cdot 65$ instead of $65+65+65+65+65$. Doing this for each data value gives the sample mean

$$
\bar{x}=\frac{1 \cdot 45+0 \cdot 55+5 \cdot 65+5 \cdot 75+9 \cdot 85+27 \cdot 95}{47} \approx 86.7
$$

On a graphing calculator, we can compute the mean of frequency data by entering the data values in L1 and the corresponding frequencies in L2:


Then compute the mean using the mean command followed by the names of the two lists.


## Practice

2. Suppose the grades on an exam for a class are recorded in the table below.

| Class | Frequency |
| :---: | :---: |
| $[25,40)$ | 2 |
| $[40,55)$ | 0 |
| $[55,70)$ | 3 |
| $[70,85)$ | 7 |
| $[85,100)$ | 8 |

Estimate the mean score on the exam using the midpoint of each class to represent the class.

Question 3 - What is the median of a dataset?

## Key Terms

Median

## Summary

If the data is arranged in numerical order, the median is the center value that splits the data into two halves.

Once the data is arranged in numerical order, the center value may be determined by dividing the total number of data by 2 . If the result of dividing the total number of data by 2 is not an integer, round the quotient up to the nearest integer. The center value is located at this position when the data is listed in numerical order.

Let's see how this is done with the data $1,2,4,6,10$. Since there are 5 data values, the median is in position 3 of the list (round up $\frac{5}{2}$ to 3 ). Since the number in position 3 is 4 , the median is 4 .
Remember to order the list in numerical order first.
If the quotient is an integer, the median is the mean of the data located in that position and the following position. For instance, the median of the data $2,4,7,12,45,45$ is the mean of the data located in position 3 and 4 . Since those data are 7 and 12, the median is $\frac{7+12}{2}$ or 9.5 .

To calculate the median of a set of data, follow the steps below.

1. Start by entering the data into a list on the calculator. Press STAT ENTER to enter a list.
2. Enter each data value under L 1 followed by ENTER.
3. Once each data value has been entered, press 2nd MODE to quit the list editor and return to the home screen.
4. From the home screen, press 2nd STAT to access the LIST commands.
5. Press $\square$ to move the cursor to the MATH menu.
6. Press $\rightarrow \square$ ENTER or 4 to paste the median ( command to the home screen.

7. After the median( command, we need to insert the name of the list we are using. Press 2nd $1 \square$ to paste the name of the list, L1 and a parentheses into the homescreen.
8. Press ENTER to calculate the median.


## Notes

## Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

| $44 \%$ | $65 \%$ | $70 \%$ | $94 \%$ | $96 \%$ | $90 \%$ | $94 \%$ | $99 \%$ | $85 \%$ | $73 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $96 \%$ | $98 \%$ | $96 \%$ | $96 \%$ | $95 \%$ | $73 \%$ | $80 \%$ | $94 \%$ | $85 \%$ | $93 \%$ |
| $96 \%$ | $73 \%$ | $94 \%$ | $67 \%$ | $61 \%$ | $93 \%$ | $97 \%$ | $80 \%$ | $99 \%$ |  |
| $81 \%$ | $61 \%$ | $87 \%$ | $83 \%$ | $96 \%$ | $95 \%$ | $84 \%$ | $69 \%$ | $92 \%$ |  |
| $91 \%$ | $90 \%$ | $78 \%$ | $88 \%$ | $97 \%$ | $90 \%$ | $97 \%$ | $91 \%$ | $97 \%$ |  |

Find the median score for the exam.
Solution To find the median of this set of data, we need to put the data in numerical order. Once this is done, divide the total number of data, 47 , by 2 . This gives us 23.5 which we round up to 24 . The median is the data value that is located in the $24^{\text {th }}$ position of the list.

A graphing calculator may be used to put the list in ascending order. Start by entering the data into list L1.


Press 2nd MODE to quit the list editor and return to the homescreen. Press 2nd STATD to access the LIST OPS menu and ENTER or 1 to paste SortA( to the homescreen. Enter the name of the list by pressing 2nd 10). Press ENTER to carry out the command. Now when you access the list by pressing STAT ENTER, the list will be shown in ascending order.


Use the arrow buttons to scroll to the $24^{\text {th }}$ entry.


The median is 91 .

## Practice

1. Suppose the grades on an exam for a class are recorded in the table below.

| $35 \%$ | $38 \%$ | $63 \%$ | $92 \%$ |
| :---: | :---: | :---: | :---: |
| $76 \%$ | $55 \%$ | $65 \%$ | $77 \%$ |
| $85 \%$ | $98 \%$ | $71 \%$ | $75 \%$ |
| $88 \%$ | $89 \%$ | $91 \%$ | $75 \%$ |
| $71 \%$ | $79 \%$ | $88 \%$ | $85 \%$ |

Find the median score for the exam.

Question 4 - What is the mode of a dataset?

## Key Terms

## Mode

## Summary

The mode of a dataset is the value that occurs most frequently in the data. To determine how frequently each data value occurs, list them in ascending order to make it easier to count the frequency of each data value.

## Notes

## Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

| $44 \%$ | $65 \%$ | $70 \%$ | $94 \%$ | $96 \%$ | $90 \%$ | $94 \%$ | $99 \%$ | $85 \%$ | $73 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $96 \%$ | $98 \%$ | $96 \%$ | $96 \%$ | $95 \%$ | $73 \%$ | $80 \%$ | $94 \%$ | $85 \%$ | $93 \%$ |
| $96 \%$ | $73 \%$ | $94 \%$ | $67 \%$ | $61 \%$ | $93 \%$ | $97 \%$ | $80 \%$ | $99 \%$ |  |
| $81 \%$ | $61 \%$ | $87 \%$ | $83 \%$ | $96 \%$ | $95 \%$ | $84 \%$ | $69 \%$ | $92 \%$ |  |
| $91 \%$ | $90 \%$ | $78 \%$ | $88 \%$ | $97 \%$ | $90 \%$ | $97 \%$ | $91 \%$ | $97 \%$ |  |

Find the mode for the exam.
Solution Examine the data carefully to determine which data value occurs most frequently. The data value 96 occurs six times in the list. Since this is the most frequently occurring data value, the mode is 96.

## Practice

1. Suppose the grades on an exam for a class are recorded in the table below.

| $35 \%$ | $38 \%$ | $63 \%$ | $92 \%$ |
| :---: | :---: | :---: | :---: |
| $76 \%$ | $55 \%$ | $65 \%$ | $77 \%$ |
| $85 \%$ | $98 \%$ | $71 \%$ | $75 \%$ |
| $88 \%$ | $89 \%$ | $91 \%$ | $75 \%$ |
| $71 \%$ | $79 \%$ | $88 \%$ | $85 \%$ |

Find the mode for the exam.

## Section 6.3 Measuring Spread

Question 1 - What is the range of the dataset?
Question 2 - What is the variance and standard deviation of a dataset?

Question 1 - What is the range of the dataset?
Key Terms
Range

## Summary

The range is the simplest way to measure how spread out a set of data is. It is calculated by taking the highest data value and subtracting the lowest data value,

$$
\text { Range }=\text { Maximum data value }- \text { Minimum data value }
$$

Notes

## Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

| 55 | 65 | 75 | 85 | 95 |
| :--- | :--- | :--- | :--- | :--- |

Find the range of the exam scores.
Solution The lowest score is 55 and the highest score is 95 . This makes the range

$$
\text { Range }=95-55=40
$$

## Practice

1. Suppose the grades on an exam for a class are recorded in the table below.

| 73 | 74 | 75 | 76 | 77 |
| :--- | :--- | :--- | :--- | :--- |

Find the range of the exam scores.

Question 2 - What is the variance and standard deviation of a dataset?

## Key Terms

Variance Standard deviation

## Summary

Variance and standard deviation are measures of how spread out a set of data is. If the variance is calculated from a population, the population variance $\sigma^{2}$ (sigma squared) of data $x_{i}$ is the mean of the squared deviations,

$$
\sigma^{2}=\frac{\sum_{i=1}^{N}\left(x_{i}-\mu\right)^{2}}{N}
$$

where $\mu$ is the population mean and $N$ is the population size.
The population standard deviation $\sigma$ is the square root of the population variance,

$$
\sigma=\sqrt{\sigma^{2}}=\sqrt{\frac{\sum_{i=1}^{N}\left(x_{i}-\mu\right)^{2}}{N}}
$$

The sample variance $s^{2}$ and sample standard deviation $s$ are calculated with similar formulas,

$$
s^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1} \quad s=\sqrt{s^{2}}=\sqrt{\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}}
$$

where $\bar{x}$ is the sample mean and $n$ is the sample size. The formulas are almost identical with $\mu$ corresponding to $x$ and $N$ corresponding to $n$. The major difference is in the denominator of the fraction. For population statistics, divide by $N$. For sample statistics, divide by $n-1$.

To calculate the standard deviation by hand, it is useful to use a table like the one below.

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^{2}$ |
| :---: | :--- | :--- |
| 35 |  |  |
| 32 |  |  |
| 41 |  |  |
| 28 |  |  |
| 31 |  |  |

The data values $35,32,41,28$, and 31 have been entered in the first column. To calculate how far each of these data lie from the mean, calculate the sample mean

$$
\bar{x}=\frac{35+32+41+28+31}{5}=33.4
$$

In the second column of the table, subtract the mean from each of the data values.

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^{2}$ |
| :---: | :---: | :---: |
| 35 | 1.6 |  |
| 32 | -1.4 |  |
| 41 | 7.6 |  |
| 28 | -5.4 |  |
| 31 | -2.4 |  |

To fill out the third column, square each of the entries in the second column and place the result in the third column.

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^{2}$ |
| :---: | :---: | :---: |
| 35 | 1.6 | 2.56 |
| 32 | -1.4 | 1.96 |
| 41 | 7.6 | 57.76 |
| 28 | -5.4 | 29.16 |
| 31 | -2.4 | 5.76 |

To calculate the sum $\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}$, sum the entries in the third column:

$$
\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}=2.56+1.96+57.76+29.16+5.76=97.2
$$

Finishing the calculation, we get

$$
s=\sqrt{\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}}=\sqrt{\frac{97.2}{5-1}} \approx 4.93
$$

On a graphing calculator, we can calculate the standard deviation by following the steps below:


## Notes

## Guided Example

Suppose the grades on an exam for a class are recorded in the table below.

| 55 | 65 | 75 | 85 | 95 |
| :--- | :--- | :--- | :--- | :--- |

a. Find the variance of the exam scores. Assume these data values are a sample of a larger set of data.

Solution Start by calculating the mean,

$$
\bar{x}=\frac{55+65+75+85+95}{5}=75
$$

Now fill out a table to help you calculate the value of $s^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}$.

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^{2}$ |
| :---: | :---: | :---: |
| 55 | -20 | 400 |
| 65 | -10 | 100 |
| 75 | 0 | 0 |
| 85 | 10 | 100 |
| 95 | 20 | 400 |

Add the numbers in the third column and divide by one less than the sample size to get the variance

$$
s^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}=\frac{1000}{4}=250
$$

b. Find the standard deviation of the exam scores. Assume these data values are a sample of a larger set of data.

Solution The standard deviation is the square root of the variance,

$$
s=\sqrt{\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n-1}}=\sqrt{250} \approx 15.81
$$

## Practice

1. Suppose the grades on an exam for a class are recorded in the table below.

| 73 | 74 | 75 | 76 | 77 |
| :--- | :--- | :--- | :--- | :--- |

a. Find the variance of the exam scores. Assume these data values are a sample of a larger set of data.
b. Find the standard deviation of the exam scores. Assume these data values are a sample of a larger set of data.

## Guided Example

Suppose an instructor records the score on an exam in a frequency table.

| Class | Frequency |
| :---: | :---: |
| $[40,50)$ | 1 |
| $[50,60)$ | 0 |
| $[60,70)$ | 5 |
| $[70,80)$ | 5 |
| $[80,90)$ | 9 |
| $[90,100)$ | 27 |

Using the midpoint of each class, estimate the standard deviation of the scores. Assume the scores are a sample.

Solution To estimate the standard deviation, we need to first calculate the mean using the frequencies,

$$
\bar{x}=\frac{1 \cdot 45+0 \cdot 55+5 \cdot 65+5 \cdot 75+9 \cdot 85+27 \cdot 95}{47} \approx 86.70
$$

When we create the table to calculate the standard deviation, we need to modify it slightly to utilize the frequencies:

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^{2}$ | $(x-\bar{x})^{2} \cdot f$ |
| :---: | :---: | :---: | :---: |
| 45 | -41.70 | 1738.89 | 1738.89 |
| 55 | -31.70 | 1004.89 | 0 |
| 65 | -21.70 | 470.89 | 2354.45 |
| 75 | -11.70 | 136.89 | 684.45 |
| 85 | -1.7 | 2.89 | 26.01 |
| 95 | 8.3 | 68.89 | 1860.03 |

The entries in the fourth column are found by multiplying the entries in the third column by the corresponding frequencies. Sum the entries in the fourth column to give

$$
\sum(x-\bar{x})^{2} \cdot f=6663.83
$$

Divide by one less than the sample size and take the square root to get the standard deviation,

$$
s=\sqrt{\frac{\sum(x-\bar{x})^{2} \cdot f}{n-1}}=\sqrt{\frac{6663.83}{46}} \approx 12.04
$$

The frequencies are utilized so that instead of carrying out the sum over every data value, we simply sum over the different values (and multiply by the corresponding frequencies.

## Practice

2. A sample of 20 college students are examined to determine the number of credit hours each student is taking. The results are summarized in the table below

| Class | Frequency |
| :---: | :---: |
| $[0,6)$ | 1 |
| $[6,12)$ | 3 |
| $[12,18)$ | 15 |
| $[18,24)$ | 1 |

Using the midpoint of each class, estimate the standard deviation of the scores. Assume the scores are a sample.

Chapter 6 Solutions

Section 6.1
Question 1
1a)

| Class | Frequency |
| :---: | :---: |
| 0 | 5 |
| 1 | 8 |
| 2 | 3 |
| 3 | 4 |

1b)

| Class | Relative <br> Frequency |
| :---: | :---: |
| 0 | 0.25 or $25 \%$ |
| 1 | 0.40 or $40 \%$ |
| 2 | 0.15 or $15 \%$ |
| 3 | 0.20 or $20 \%$ |

1c) Based on the relative frequency table, the percentage of hours with 1 or less defective units is $25 \%+40 \%$ or $65 \%$.

2a)

| Class | Frequency |
| :---: | :---: |
| $[25,40)$ | 2 |
| $[40,55)$ | 0 |
| $[55,70)$ | 3 |
| $[70,85)$ | 7 |
| $[85,100)$ | 8 |

2b)

| Class | Relative <br> Frequency |
| :---: | :---: |
| $[25,40)$ | 0.10 or $10 \%$ |
| $[40,55)$ | 0 or $0 \%$ |
| $[55,70)$ | 0.15 or $15 \%$ |
| $[70,85)$ | 0.35 or $35 \%$ |
| $[85,100)$ | 0.40 or $40 \%$ |

2c) $10 \%+15 \%$ or $25 \%$
Question 2
1)


Question 3
1)


Section 6.2
Question 2 1) 1.3, 2) 76.75
Question 3 1) 76.5
Question 4 1) $71,75,85$, and 88 all occur twice (most frequently) so there is no mode

Section 6.3
Question 1 1) 4
Question 2 1) 2.5 2) approximately 3.69

Section 6.4

