8.1 Permutations

Question 1: How do you count choices using the Multiplication Principle?

Question 2: What is factorial notation?

Question 3: What is a permutation?

Question 1: How do you count choices using the Multiplication Principle?

Key Terms

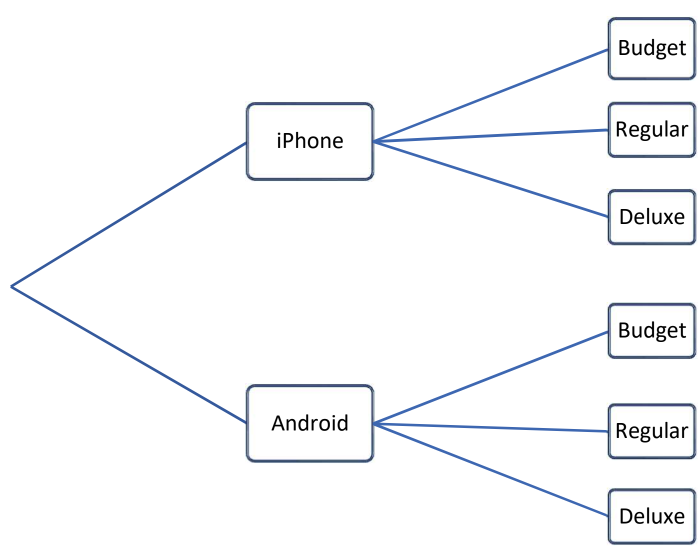
Multiplication Principle

Summary

Businesses are often faced with decisions in which they may make many choices. To count the number of ways these decisions are made, we can apply the **Multiplication Principle**. Let's see where this principle comes from through an example.

A small cellular provider gives its customers two choices of phones to use. They may use an iPhone or a phone that uses the Android operating system. In addition, the company offers three different calling plans: Budget plan, Regular plan, and the Deluxe plan. How many different choices of phone and calling plan does a customer have?

To answer this question, we can use a decision tree and list out the choices a customer may make in a decision tree.



A decision tree shows the different choices a customer makes when choosing a phone and plan. If we move left to right through the tree, we can list out each of the possibilities. The first set of branches lists the choice of phones and the second set of branches lists the plans:

iPhone with Budget plan	iPhone with Regular plan	iPhone with Deluxe plan
Android with Budget plan	Android with Regular plan	Android with Deluxe plan

By listing out each of the possibilities, we see that there are six possible phone/plan choices. The decision tree helps us to list out these possibilities. However, if we only need to know how many choices, we can multiply the number of choices for phones and plans.

$$\underline{\;\;2\;\;} \cdot \underline{\;\;3\;\;} = 6$$

Number of phones to choose from

Number of plans to choose from

This strategy is useful for determining the total number of choices even when there are a larger number of choices.

Multiplication Principle

Suppose we wish to know the number of ways to make *n* choices where there are

d_1 ways to make choice 1

d_2 ways to make choice 2

$\vdots$

d_n ways to make choice n

Then the total number of ways to make all of the choices is

$$d_1 \cdot d_2 \cdot \ldots \cdot d_n$$

If we can enumerate the number of ways to make a series of decisions, the product of these numbers tells us how many ways there are to make this series of decision.

Notes

Guided Example 1	Practice
An online custom bicycle seller wishes to count the total number of different types of bicycles that are available through its website. The seller offers 4 different frame styles, 8 different fender colors, 10 different tire colors, 8 different wheel colors, 6 different pedal colors, and 12 different accessory colors. How many different bicycles can a customer order? **Solution** Each choice the customer must make leads to a different factor in the multiplication principle. $\underbrace{4}_{\text{frame style}} \cdot \underbrace{8}_{\text{fender color}} \cdot \underbrace{10}_{\text{tire color}} \cdot \underbrace{8}_{\text{wheel color}} \cdot \underbrace{6}_{\text{pedal color}} \cdot \underbrace{12}_{\text{accessory color}} = 184{,}320$ There are 184,320 different bicycles that can be ordered.	The owner of a Great Purchase, an online stereo store, wants to advertise that he has many different sound systems in stock. The store carries 6 different Blu Ray players, 10 different receivers, and 5 different speakers. Assuming a sound system consists of one of each, how many different sound systems can he advertise?

Guided Example 2

A company wants to have 3-digit phone extensions with the first digit not being zero.

a. How many possible extensions are there?

Solution Break the problem into three choices that need to be made. These choices indicate the number of ways to choose each of the three numbers in the extension. Since the first digit cannot be zero, there are 9 ways to choose the first number. The second and third numbers may contain zero, so there are 10 ways to choose each of those numbers. The total number of extensions is the product of these choices,

$$\underset{\substack{\text{first}\\\text{digit}}}{\underline{9}} \cdot \underset{\substack{\text{second}\\\text{digit}}}{\underline{10}} \cdot \underset{\substack{\text{third}\\\text{digit}}}{\underline{10}} = 900$$

b. For moral and safety reasons, the company wants to exclude the extensions 911 and 666. How many possible 3-digit extensions are there without these exclusions?

Solution There are 900 extensions that exclude 0 as the first digit. The extensions 911 and 666 are two specific extensions. Excluding those extensions leaves 898 extensions.

c. How many possible 3-digit extensions are there if the first digit is not zero and digits may not be repeated?

Solution When digits are not repeated, the number of choices for the second and third digits is reduced:

$$\underset{\substack{\text{first}\\\text{digit}}}{\underline{9}} \cdot \underset{\substack{\text{second}\\\text{digit}}}{\underline{9}} \cdot \underset{\substack{\text{third}\\\text{digit}}}{\underline{8}} = 648$$

For the second digit, we may now use a zero, but not the first digit. This yields 9 choices. For the third digit, we cannot use either of the first two digits giving 8 possible digits.

Practice

A pin code consists of four digits from 0 to 9 at a bank.

a. How many possible pin codes are there?

b. How any possible pin codes are there if the digits may not be repeated?

Question 2: What is factorial notation?

Key Terms

Factorial

Summary

When we apply the Multiplication Principle and do not allow repetition, the number of choices in each part of the product drops by 1. This leads to products like $5\cdot 4\cdot 3\cdot 2\cdot 1$ or $8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$.

This type of product occurs so often that it is assigned its own symbol.

Factorial Notation

For any positive integer *n*,

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$

The value of $0!$ is defined to be 1.

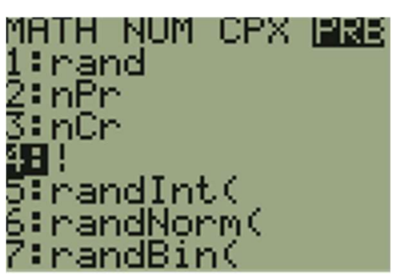
Let's look at how we might apply this to an application.

Suppose a production line requires six workers to carry out six different jobs. Each worker can only do one job at a time. Once a worker is selected for a job, the other jobs must be carried out by the remaining workers. To find the number of ways we can assign workers to jobs, calculate the product

$$\underset{\substack{\text{first}\\\text{job}}}{\underline{6}}\cdot\underset{\substack{\text{second}\\\text{job}}}{\underline{5}}\cdot\underset{\substack{\text{third}\\\text{job}}}{\underline{4}}\cdot\underset{\substack{\text{fourth}\\\text{job}}}{\underline{3}}\cdot\underset{\substack{\text{fifth}\\\text{job}}}{\underline{2}}\cdot\underset{\substack{\text{sixth}\\\text{job}}}{\underline{1}} = 720$$

The number of ways to make each choice drops by one in each factor since each worker can only do one job. In effect, we can't choose the same worker twice. This is often indicated by saying that we want to assign workers without repetition.

Instead of multiplying these factors out, we can utilize factorials and write it as $6!$. This may then be computed on a calculator such as a TI graphing calculator. The factorial symbol is located under the MATH button in the PRB submenu.



Notes

Guided Example 3

Evaluate each of the expressions involving factorials.

a. 8!

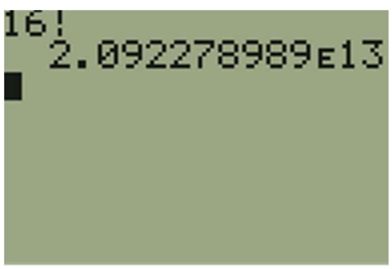
Solution Using the definition of factorial, this is equal to

$$8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$$

b. 16!

Solution Using the definition of factorial, this is equal to

$$16 \cdot 15 \cdot 14 \cdots 3 \cdot 2 \cdot 1 \approx 2.092 \times 10^{13}$$



This is an incredibly huge number. The calculator uses scientific notation to display it. The E13 indicates $\times 10^{13}$. You would write this down by moving the decimal place 13 places to the right.

Practice

Evaluate each of the expressions involving factorials.

a. 5!

b. 20!

c. $\frac{100!}{98!}$ **Solution** Both of the numbers in the fraction are beyond most calculator's ability to calculate. However, if we use the definition of factorial we note an interesting pattern. $$\frac{100!}{98!} = \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}{98 \cdot 97 \cdots 3 \cdot 2 \cdot 1}$$ Many of the factors on top match up with identical factors on the bottom. These may be reduced leaving $$\frac{100!}{98!} = 100 \cdot 99 = 9900$$	c. $\frac{82!}{80!}$

Question 3: What is a permutation?

Key Terms

Permutation

Summary

The term "**permutation**" refers to different arrangements of objects. We have already seen on example of a permutation in Question 2. When we allocated six workers among six jobs on a production line, we were counting the number of ways that we could arrange the workers among the jobs. This was a permutation of 6 workers taken 6 at a time. For permutations, we assume that the objects are arranged without repetition. In the context of the production line, this means that once a worker is given a job that worker cannot be assigned to another job.

How many ways can we assign six workers to four jobs without repetition? As before, we need to choose workers for each job.

$$\underset{\text{first job}}{\underline{6}} \cdot \underset{\text{second job}}{\underline{5}} \cdot \underset{\text{third job}}{\underline{4}} \cdot \underset{\text{fourth job}}{\underline{3}} = 360$$

This is the same pattern that resulted in factorial notation, but we are missing the last two factors. However, we can still write this product in terms of factorial notation.

$$6 \cdot 5 \cdot 4 \cdot 3 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = \frac{6!}{2!} = \frac{6!}{(6-4)!}$$

The number in the numerator indicates the number of objects we are selecting from. The number in the denominator is the difference between the number of objects we are selecting from and the number we are selecting. This relationship leads to a general rule for permutations.

Permutations

An arrangement of n objects taken r at a time without repetition is called a permutation. The number of these arrangements is symbolized $P(n,r)$ and found with

$$P(n,r) = \frac{n!}{(n-r)!}$$

where $r \le n$. The symbol $P(n,r)$ is read "the permutation of n objects taken r at a time".

It can be confusing to use the letter *P* to indicate permutations and probability. For this reason, some textbooks will write $P_{n,r}$ or P_r^n instead of $P(n,r)$. In practice, this is less of a concern since the values in parentheses are numbers for permutations and events represented by capital letters in probability.

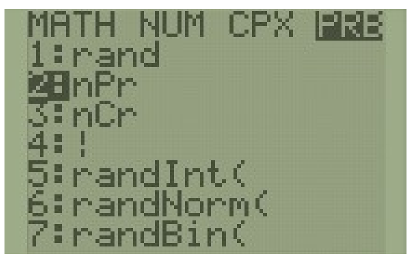
Permutation assume different arrangements of objects are counted separately. For instance, we could list the permutations of the letters CAT taken two at a time:

CA	AT	CT
AC	TA	TC

For permutations, order makes a difference, so CA is counted separately from AC. The number of permutations of three letters taken two at a time is $P(3,2)$ and is calculated as

$$P(3,2) = \frac{3!}{(3-2)!} = 6$$

Permutations may be calculated on a TI graphing calculator by entering the number of objects, nPr (press $\boxed{\text{MATH}}$ and goto PRB submenu), and how many are taken at a time.



Notes

Guided Example 4

Evaluate each of the expressions below.

a. $P(10,5)$

Solution

$$P(10,5) = \frac{10!}{(10-5)!} = 30240$$

b. $P(8,1)$

Solution

$$P(8,1) = \frac{8!}{(8-1)!} = 8$$

Practice

Evaluate each of the expressions below.

a. $P(9,4)$

b. $P(6,1)$

Guided Example 5	Practice
The US Postal Service has used 5-digit zip codes since 1963 to help it to deliver mail in the US. a. How many zip codes are possible if there are no restrictions on the digits used? **Solution** Each digit can be any integer from 0 to 9. Since digits may be repeated, we need to use the Multiplication Principle to find the number of ways the numbers may be arranged: $\underset{\text{first digit}}{\underline{10}} \cdot \underset{\text{second digit}}{\underline{10}} \cdot \underset{\text{third digit}}{\underline{10}} \cdot \underset{\text{fourth digit}}{\underline{10}} \cdot \underset{\text{fifth digit}}{\underline{10}} = 10,000$ b. How many zip codes are possible if digits may not be repeated? **Solution** When digits are not repeated, we may use permutation to count the number of ways to select 5 digits from 10 digits: $P(10,5) = \frac{10!}{(10-5)!} = 30240$ Note that a permutation with P requires that we cannot repeat a selection.	In 1983, The US Postal Service introduced ZIP + 4 so that codes now consisted of 9 digits. a. How many zip codes are possible if there are no restrictions on the digits used? b. How many zip codes are possible if digits may not be repeated?

Guided Example 6

A city council consists of seven members. Two of those members are chosen to be mayor and vice mayor. In how many ways can this be done?

Solution We are interested in calculating the number of ways to arrange two council members taken two at a time. Since a council member cannot be mayor and vice mayor, repetition is not allowed. The number of arrangements is

$$P(7,2)=\frac{7!}{(7-2)!}=42$$

Practice

A business wants to select three employees to honor. One employee will be named employee of the year, another will be named volunteer of the year, and the last will be named advocate of the year. If there are 20 employees in the company and employees can only be selected for one honor, how many ways are there to honor employees?

8.2 Combinations

Question 1: What is a combination?

Question 2: What is the difference between a permutation and combination?

Question 1: What is a combination?

Key Terms

Combinations

Summary

In Section 8.1, we used permutations to count different arrangements of objects. The word "arrangements" is used since different orders of objects must be counted separately. Different arrangements of numbers and letters on an auto license plate lead to different license plate numbers.

In many applications, different arrangements of objects are not counted differently. Many states have lotteries that are used to fund schools and environmental causes. In these lotteries, lottery officials select numbered balls from a group of balls labeled 1 through a larger number like 52. A player wins the lottery jackpot if the numbers the player selects matches the numbers on the balls selected by officials. The order in which the balls are drawn does not have to be duplicated. The number of ways the balls can be selected from a larger group of balls is calculated using **combinations**. In combinations, groupings of objects are counted and the order of the objects in the grouping are irrelevant.

Combinations

A combination of n objects selected r at a time is a group of r objects selected from n different objects without regard to order. The number of combinations is

$$C(n,r) = \frac{n!}{(n-r)!\,r!}$$

For the same values of n and r, the number of combinations is always smaller than the number of permutations due to the $r!$ factor in the denominator. This makes sense since combinations do not count the rearrangements of the letters, only the different groupings.

For example, in the last section we looked at the permutations of the letters CAT taken two at a time:

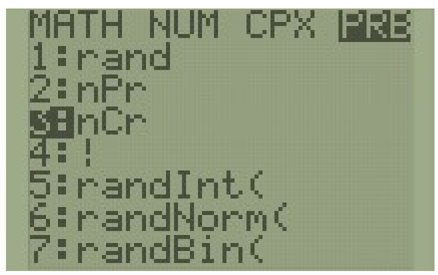
CA	AT	CT
AC	TA	TC

Since AC and CA (as well as AT and TA, CT and TC) are counted separately, the order of the letters makes a difference. In combinations, AC and CA are counted as one since order does not make a difference. So, the combinations of the letters CAT taken two at a time is

CA	AT	CT

The number of combinations would be calculated as $C(3,2) = \dfrac{3!}{(3-2)\,!2!} = 3$.

Combination may be computed on a TI graphing calculator using the MATH button and then selecting the PRB submenu.



The third selection will compute a combination when the number of objects is entered in front of nCr and the number of objects selected is entered after the command.

<u>Notes</u>

Guided Example 1

Evaluate the expressions below.

a. $C(7,5)$

Solution

$$C(7,5) = \frac{7!}{(7-5)!\,5!} = 21$$

b. $C(10,9)$

Solution

$$C(10,9) = \frac{10!}{(10-9)!\,1!} = 10$$

Practice

Evaluate the expressions below.

a. $C(6,3)$

b. $C(8,7)$

Guided Example 2	Practice
A small school has 4 teachers and 50 students. a. How many different committees can be formed from teachers and students if the committee consists of five people? **Solution** When counting committees where there are no assigned duties, rearranging committee members yields the same committee. Combinations are used to count the number of committees. Since there is a total of 54 teachers and students, the number of ways to select 5 from the total of 54 is $C(54,5)=\frac{54!}{(54-5)!5!}=3,162,510$ This counts the number of ways to grab groups of 5 from a total of 54 people where the order of the people in the group is irrelevant.	From a group of 10 smokers and 20 nonsmokers, a researcher wants to randomly select smokers and nonsmokers for a study. a. How many ways is there to select 5 smokers for the study?
b. How many different committees can be formed from the students if the committee must have five students? **Solution** There are 50 students and we wish to find how many ways there is to select groups of 5. Ordering is irrelevant so we calculate with combinations, $C(50,5)=\frac{50!}{(50-5)!5!}=2,118,760$	b. How many ways is there to select 5 nonsmokers for the study?
c. How many committees of students and teachers may be formed if the committee must have 2 teachers and 3 students? **Solution** Start by breaking this down into two tasks: count the number of groupings of two teachers from the total of 4 and count the number of groupings of 3 students from the total of 50. Since order does not matter, we can compute each of these with combinations:	c. How many ways is there to select 5 smokers and 5 nonsmokers for the study?

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$$C(4,2) = \frac{4!}{(4-2)!2!} = 6$$

$$C(50,3) = \frac{50!}{(50-3)!3!} = 19,600$$

Because of the Multiplication Principle, the total number of ways to order teachers and students for the committee is the product of these numbers,

$$C(4,2) \cdot C(50,3) = 6 \cdot 19,600 = 117,600$$

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<th>Guided Example 3</th>
<th>Practice</th>
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A state lottery game requires that you pick 5 different numbers from 1 to 54.

a. If you pick of the numbers correctly, you win \$250,000. In how many ways can you pick the winning numbers without regard to order?

Solution The order in which lottery numbers are picked does not matter. To win the prize, you must match five numbers out of five numbers without regard to order. The number of ways to do this is

$$C(5,5) = \frac{5!}{(5-5)!5!} = 1$$

b. A lesser prize is won if you select 4 of the winning numbers out of 5. In how many ways can you pick lesser winning numbers without regard to order?

Solution You need to pick 4 of the winning numbers along with a non-winning number. The number of ways to pick the winning numbers is

$$C(5,4) = \frac{5!}{(5-4)!4!} = 5$$

If 5 numbers are winning numbers, the other 49 must be losing numbers. The number of ways to pick the lesser prize is

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A state lottery game requires that you pick 4 different numbers from 1 to 36.

a. If you pick of the numbers correctly, you win \$50,000. In how many ways can you pick exactly of the winning numbers without regard to order?

b. A lesser prize is won if you select 3 of the winning numbers out of 4. In how many ways can you pick lesser winning numbers without regard to order?

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$$C(5,4)\cdot 49 = 5\cdot 49 = 245$$ c. How many ways are there to pick 5 different lottery numbers from 1 to 54 without regard to order? **Solution** From the 54 numbers, we want to pick a group of 5. Using combinations, we get $$C(54,5)=\frac{54!}{(54-5)!5!}=3{,}162{,}510$$	c. How many ways are there to pick 4 different lottery numbers from 1 to 36 without regard to order?

Question 2: What is the difference between a permutation and combination?

Key Terms

Summary

We have looked at several ways of counting. Each counting strategy is applicable under certain assumptions. The table below outlines the assumptions and formulas for each strategy.

Strategy	Multiplication Principle	Permutations	Combinations
Purpose	Number of ways to make n choices where there are d_i ways to make i^{th} choice	Number of ways to select r objects from n different objects	
Repetition Allowed?	Yes	No	No
Order Important?	Yes	Yes	No
Formula	$d_1 \cdot d_2 \cdot \ldots \cdot d_n$	$P(n,r) = \dfrac{n!}{(n-r)!}$	$C(n,r) = \dfrac{n!}{(n-r)!\,r!}$

Sone problems may require more than one strategy. In those situations, it helps to break the problem into several choices, count the number of ways the choices may be made, and then multiply the choices using the Multiplication Principle.

Notes

Guided Example 4	Practice
A group of 15 workers decides to send a delegation of 4 to their supervisor to discuss their work assignments. In the following cases, determine the possible number of delegations. a. The number of possible 4 person delegations. **Solution** In the delegation sent to the supervisor, there are no assigned roles so the ordering are irrelevant, In addition, each person may only ne sent once so repetition is not allowed. Using combinations, the number of possible delegations is $$C(15,4)=\frac{15!}{(15-4)!4!}=1365$$ b. The number of 4 person delegations that include the foreman. **Solution** Since the foreman is a required member of the delegation, only three remaining members will be selected from the 14 remaining workers. The number of possible delegations is $$C(14,3)=\frac{14!}{(14-3)!3!}=364$$ c. Suppose there are 7 women and 8 men in the group. How many delegations include exactly one woman? **Solution** Since repetition is not allowed and order makes no difference, we will still use combinations to count the delegations. However, we need to break the delegation into two decisions and apply the Multiplication Principle: $$\underbrace{C(7,1)}_{\text{choose woman}}\cdot\underbrace{C(8,4)}_{\text{choose men}}=7\cdot 70=490$$	A town is forming a committee to investigate ways to improve water conservation in the town. a. In how many ways can a committee of 8 be selected from a group of 29 people? b. To make sure all constituencies are represented, the committee will consist of 2 members from the 7-person town council, 3 members of a 10-person citizens advisory board, and 3 members of the town's 12-person utility department. How many ways can that committee be formed?

Guided Example 5

Licenses plates on automobiles consists of letters and numbers where the letters and numbers may be repeated. a. How many licenses plates are there with three letters followed by three numbers? **Solution** In this situation, different arrangements of letters yield different license plates, so order makes a difference. Additionally, the letters and numbers may be repeated. Because of these facts, we'll apply the Multiplication Principle where each decision regards a letter or number: $\underset{\text{first letter}}{\underline{26}} \cdot \underset{\text{second letter}}{\underline{26}} \cdot \underset{\text{third letter}}{\underline{26}} \cdot \underset{\text{first number}}{\underline{10}} \cdot \underset{\text{second number}}{\underline{10}} \cdot \underset{\text{third number}}{\underline{10}} = 17,576,000$ There are a total of 17,576,000 license plates with three letters followed by three numbers.	a. How many different three-digit numbers can be formed using the digits 1, 3, 5, 7, and 9 without repetition? For example, 911 is not allowed.
b. How many license plates are the with six letters or numbers? **Solution** In this case, any of the entries on the license plate may be a letter of number. Applying the Multiplication Principle yields $\underset{\text{first entry}}{\underline{36}} \cdot \underset{\text{second entry}}{\underline{36}} \cdot \underset{\text{third entry}}{\underline{36}} \cdot \underset{\text{fourth entry}}{\underline{36}} \cdot \underset{\text{fifth entry}}{\underline{36}} \cdot \underset{\text{sixth entry}}{\underline{36}} = 2,176,782,336$	b. How many different two-digit numbers can be formed using the digits 2, 4, 5, 6, and 1 with repetition? For example, 11 is allowed.

Guided Example 6

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An engineering firm with 15 employees is sending a five-person team to deliver a proposal. The team must have a team leader and a main presenter; the other members have no particularly defined roles. In how many ways can this team be formed?

Solution Break the team into two parts: the two people with defined roles and the three people without defined roles.

Rearranging the three roles would lead to different teams so order makes a difference for that part. Assuming no member can have more than one role, the number of ways to assign the team leader, main presenter is

$$P(15,2)=\frac{15!}{(15-2)!}=210$$
Since the remaining member have no defined roles, order does not matter and combinations may be used to compute the number of ways to select three members from the remaining 13 employees,

$$C(13,3)=\frac{13!}{(13-3)!3!}=286$$
Because of the Multiplication Principle, these numbers multiply, $210\cdot 286$, to give 60,060 possible teams.

Note that this problem could also be solve by selecting the members with no defined roles followed tb the team leader and presenter,

$$C(15,3)\cdot P(12,2)=455\cdot 132=60,060$$
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The PTK Honors program has 20 members. The program’s board has a president, vice president, treasurer and two members at large (all of whom must be different).

a. In how many ways can the board be selected?

b. The program wants to select two board members to manage a food bank. In how many ways can these board members be selected?
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8.3 Probability with Permutations and Combinations

Question 1: How do you find the likelihood of a certain type of license plate?

Question 2: How do you find the likelihood of a particular committee?

Question 3: How do you find the probability of winning a lottery?

Question 4: How do you find the likelihood of detecting a defective product?

Question 1: How do you find the likelihood of a certain type of license plate?

Key Terms

Summary

To find the likelihood of an event E with equally likely outcomes, we need to count the number of outcomes in the event and divide it by the total number of outcomes in the sample space.

$$P(E) = \frac{n(E)}{n(S)}$$

For this question, the events involve counting license plates. Since the order of the letters and numbers on a license plate matters, we typically use the Multiplication Principle to calculate the number of outcomes in the event and sample space. However, if repetition of letters or numbers are not allowed, we can also use permutations.

Notes

Guided Example 1	Practice
If the license plates in a particular state consist of three letters followed by three numbers, what is the probability that a randomly generated plate begins with the letters "NUT"? **Solution** Since the first three letters must be "NUT", the only choices to made are the numbers. Using the Multiplication Principle yields, $\underset{\text{first number}}{\underline{10}} \cdot \underset{\text{second number}}{\underline{10}} \cdot \underset{\text{third number}}{\underline{10}} = 1000$ To find the total number of license plates that have three letters followed by three numbers, apply the Multiplication Principle again: $\underset{\text{first letter}}{\underline{26}} \cdot \underset{\text{second letter}}{\underline{26}} \cdot \underset{\text{third letter}}{\underline{26}} \cdot \underset{\text{first number}}{\underline{10}} \cdot \underset{\text{second number}}{\underline{10}} \cdot \underset{\text{third number}}{\underline{10}} = 17,576,000$ Dividing these numbers gives thee likelihood that a randomly generated license plate would start with "NUT", $P(\text{starts with NUT}) = \dfrac{n(\text{starts with nut})}{n(\text{3 letters, 3 numbers})} = \dfrac{1000}{17,576,000} \approx 0.000057$ Or approximately 0.0057%.	If the license plates in a particular state consist of four letters followed by three numbers, what is the probability that a randomly generated plate ends with the numbers "666"?

Question 2: How do you find the likelihood of a particular committee?

Key Terms

Summary

As we saw earlier, committees with no defined roles are counted with combinations since order does not matter and repetition is no allowed. However, if roles are defined in the committee like president or vice president, permutations must be used to count the committee.

Notes

Guided Example 2	Practice
A Congressional committee consists of 8 men and 10 women. A subcommittee of 4 people is set up at random. What is the probability that it will consist of all men? **Solution** Let's start by determining how many subcommittees of four that may be selected from 18 people. Since this is a committee with no assigned roles, combinations give us $$C(18,4)=\frac{18!}{(18-4)!4!}=3060$$ This gives us the number of outcomes in the sample space where the sample space is all subcommittees of 4 selected from 18 members. The event we are interested in is all subcommittee members are males. Since there are 8 men in the committee, the number of 4-person male subcommittees is $$C(8,4)=\frac{8!}{(8-4)!4!}=70$$ The likelihood of an all-male committee is $$P(\text{all male subcommittee})=\frac{70}{3060}\approx 0.0229$$ or about 2.29%.	A Congressional committee consists of 12 men and 6 women. A subcommittee of 4 people is set up at random. What is the probability that it will consist of all women?

Question 3: How do you find the probability of winning a lottery?

Key Terms

Lotteries are contests in which numbers are randomly selected from a group. If the numbers a player chooses matches the numbers chosen by the organization conducting the lottery, the player wins a prize. Lesser prizes ma be awarded for matching fewer numbers.

In “The Pick” conducted by the state of Arizona, the state randomly picks six numbers from the numbers 1 through 44. If the numbers the player picks match the states numbers, the player wins the jackpot. Smaller prizes are awarded for matching five, four or three numbers. Since the order in which numbers are picked are irrelevant, combinations are used to find the number of ways to pick numbers.

Summary

Notes

Guided Example 3	Practice
A state lottery game requires that you pick 6 different numbers from 1 to 44. What is the probability of picking exactly 3 of the 6 numbers correctly? **Solution** To determine the number of ways the state can pick numbers, we find the combination of six numbers selected from 44, $$C(44,6)=\frac{44!}{(44-6)!6!}=7,059,052$$ To pick exactly three numbers correctly, we need three winning numbers and three losing numbers. The number of ways to do this is $$C(6,3)\cdot C(38,3)=20\cdot 8436=168,720$$ The likelihood of matching three of the winning numbers is $$P(\text{matching 3 numbers})=\frac{168,720}{7,059,052}\approx 0.0239$$ or about 2.39%.	A state lottery game requires that you pick 6 different numbers from 1 to 52. What is the probability of picking exactly 4 of the 6 numbers correctly?

Question 4: How do you find the likelihood of detecting a defective product?

Key Terms

Summary

When checking inventory for defective products, a manufacturer is not interested in how the defective units are ordered. They are interested in how many defective items are in a sample. For this reason, combinations are used to count the number of ways defective items may be picked from a sample.

Notes

Guided Example 4	Practice
A shipment of 20 cars contains 3 defective cars. Find the probability that a sample of size 2, drawn from the 20, will not contain exactly one defective car. **Solution** Start by determining how many ways there are to select a sample of 2 cars from 20 cars. Since order does not matter and repetition is not allowed, combinations are used to calculate the number of ways to select a sample of 2 from 20 cars, $$C(20,2)=\frac{20!}{(20-2)!2!}=190$$ For the sample to contain exactly one defective car, a car must be selected from the defective cars and another from the non-defective cars. The number of ways this can be done is $$\underbrace{C(3,1)}_{\text{defective}}\cdot\underbrace{C(17,1)}_{\text{non-defective}}=3\cdot 17=51$$ By dividing the number of ways to pick exactly one defective by the number of ways to select a sample of 2 from 20, we get the likelihood of getting exactly one defective car, $$P(\text{exactly one defective car})=\frac{51}{190}\approx 0.268$$ or approximately 26.8%. .	A bag of 40 Cadbury Easter Eggs contains 2 defective eggs. Find the probability that a sample of size 3, drawn from the 40, will contain exactly one defective egg.