Find where $f(x) = x e^{-x^2}$ *is increasing.*

To solve this problem, we'll need to construct a number line that tracks the behavior of the derivative $f'(x)$. Since we are looking for where the function is increasing, we'll look for where $f'(x) > 0$.

Take the derivative of f(x): Start by using the product rule with

$$u = x$$
$$v = e^{-x^2}$$

The derivative of u is 1. To find v', we'll need to use the chain rule to get

$$v' = e^{-x^2} \cdot (-2x)$$

Using these derivatives in the product rule gives us

$$f'(x) = e^{-x^2} \cdot 1 + x \cdot e^{x^2} \cdot (-2x)$$

To make this easier to work with, factor out the e^{-x^2}:

$$f'(x) = e^{-x^2}\left(1 - 2x^2\right)$$

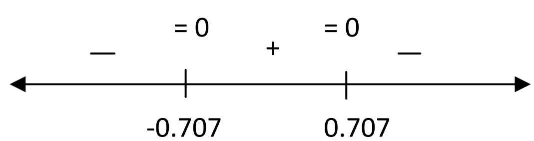
Find the critical number: To find the critical number, we need to set the derivative equal to zero. Since the derivative is written as a product, the derivative will be zero when either one of the pieces is zero:

$$e^{-x^2} = 0 \quad \text{or} \quad 1 - 2x^2 = 0$$

Exponential functions are always positive, so the first piece of the product can never be zero. We can solve the second piece to get

$$\begin{aligned} 1 - 2x^2 &= 0 \\ 1 &= 2x^2 \\ \tfrac{1}{2} &= x^2 \\ x &= \pm\sqrt{\tfrac{1}{2}} \approx \pm 0.707 \end{aligned}$$

Make a number line to track the derivative: Put the critical numbers on a number line and test the derivative on either side of the critical numbers to find where it is positive.



This easiest way to do this is to put the values -1, 0 and 1 into $f'(x)$.

Interpret the number line: The function is increasing where the derivative is positive. According to the number line, this is between the critical points or on $\left(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}}\right)$.