The speed of a particle in a test laboratory was noted every half second for 2 seconds. The results are shown in the table below. Make an overestimate and an underestimate of the total distance traveled by the particle in the first two seconds.

t (sec)	0	.5	1.0	1.5	2.0
v (feet per sec)	10	6.5	6	5.5	5.3

By looking at each half second, we can determine the shortest and longest distance the particle could travel. Add up all of the overestimates and all of the underestimates to get an overestimate and an underestimate of the total distance traveled.

Estimate the distance the particle traveled over the first half second: Assuming the particle's speed is decreasing over the entire interval from t = 0 to t = .5, the highest speed it traveled is 10 feet per second and the slowest speed it traveled is 6.5 feet per second. This means that the farthest it could have traveled in the first half second is (10 feet per second)(.5 second) = 5 feet. The least it could have traveled is (6.5 feet per second)(.5 second) = 3.25 feet.

Estimate the distance the particle traveled over the second half second: Using the same reasoning for the second half second, the farthest the particle could go is (6.5 feet per second)(.5 second) = 3.25 feet. The least distance would be (6 feet per second)(.5 second) = 3 feet.

Estimate the distance the particle traveled from t = 1.0 to t = 1.5: The farthest the particle could go is (6 feet per second)(.5 second) = 3 feet. The least distance is (5.5 feet per second)(.5 second) = 2.75 feet.

Estimate the distance the particle traveled from t = 1.5 to t = 2.0: The farthest the particle could go is (5.5 feet per second)(.5 second) = 2.75 feet. The least distance is (5.3 feet per second)(.5 second) = 2.65 feet.

Find the estimates of the total distance traveled from t = 0 to t = 2: By adding all of the overestimates we get

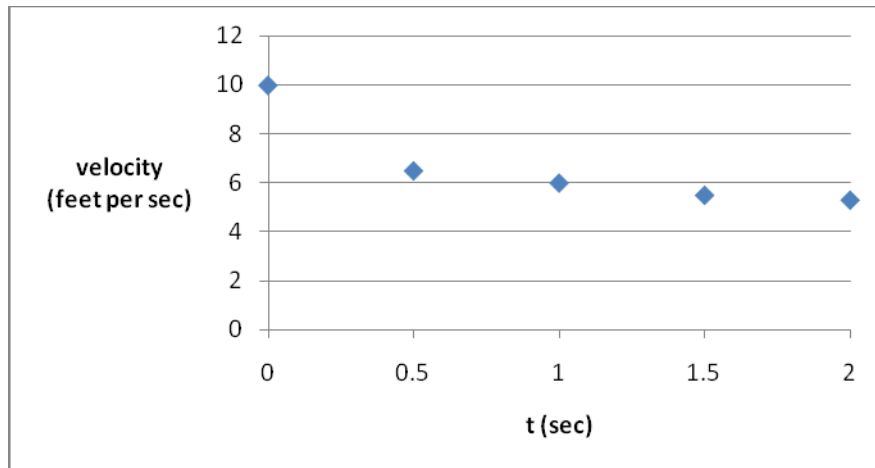overestimate of total distance = (10)(.5) + (6.5)(.5) + (6)(.5) + (5.5)(.5) = 14 feet
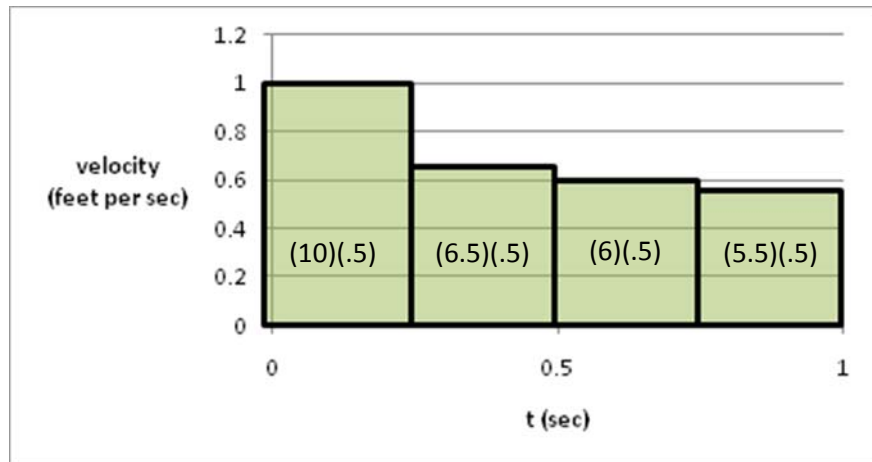
Adding the underestimates gives

underestimate of total distance = (6.5)(.5) + (6)(.5) + (5.5)(.5) + (5.3)(.5) = 11.65 feet

The actual distance traveled will be somewhere between these values depending on how the particles speed changes over the first two seconds.

A graphical interpretation: Let's look at a scatter plot of the data.

We can use this graph to generate the terms in the overestimate of the total distance traveled above. To do this we'll graph some strategically placed rectangles. Over each half second interval, graph the base of the rectangle. Draw the rectangle so that the top of the rectangle touches the data point on the left of the interval. The area of each rectangle corresponds to the terms in the overestimate. This sum is called a left hand estimate since the heights on the rectangle are determined from the point on the left hand side of the interval.

The same idea applies to the underestimate. In this case, we draw the rectangles with the top touching the right hand point on the interval. The area of these rectangles corresponds to a right hand estimate. Notice that the height of each rectangle is in feet per second and the width is in seconds. The product corresponds to the area of the rectangle and has units

$$\frac{\text{feet}}{\text{second}} \cdot \text{second} = \text{feet}$$