Find the point of diminishing returns (x, y) for the function

$$R(x) = 10,000 - x^3 + 42x^2 + 800x, \quad 0 \le x \le 20$$

where $R(x)$ represents the revenue (in thousands of dollars) and x represents the amount spent on advertising (in thousands of dollars).

The point of diminishing returns is simply the point of inflection on this function. To find it, we'll need to find where the second derivative of $R(x)$ is equal to 0 or does not exist. Once we have located these values, we'll make a number line to track the sign of the second derivative to insure that it changes signs.

Find the Second Derivative

To take the derivative of $R(x)$, apply the power rule to the terms:

$$R'(x) = -3x^2 + 84x + 800$$

Now apply the power rule again to obtain

$$R''(x) = -6x + 84$$

Make the Second Derivative Number Line

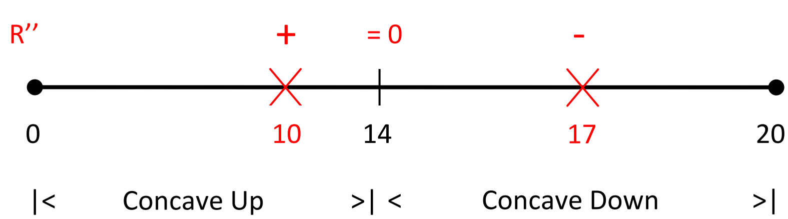
Set the second derivative equal to zero and solve for x:

$$-6x + 84 = 0$$

$$-6x = -84$$

$$x = \frac{84}{6} \text{ or } 14$$

We'll place this value on a number line from 0 to 20 (where the function is defined) and text values to see if the concavity of the function changes.



Since the concavity changes at $x = 14$, there is a point of inflection there. To find the point of diminishing returns, we need to find the value of the revenue function at $x = 14$:

$$R(14) = 10,000 - 14^3 + 42(14)^2 + 800(14) = 26,688$$

This means when $14,000 are spent on advertising, the revenue is $26,688,000.

Prior to $x = 14$, each dollar spent on advertising leads to higher and higher revenue since the revenue function is getting steeper and steeper. Beyond $x = 14$, the revenue is still increase but is getting less steep. This means that beyond the point of diminishing returns, there is a smaller increase in the revenue for each dollar invested in advertising.