

Reduced Row Echelon Form in Excel

In this example, we want to utilize Excel to solve the system

$$4x - 2y - 5z = 11$$

$$x + y + z = 2$$

$$-2x + 3y - 2z = -14$$

Start by converting this system to an augmented matrix,

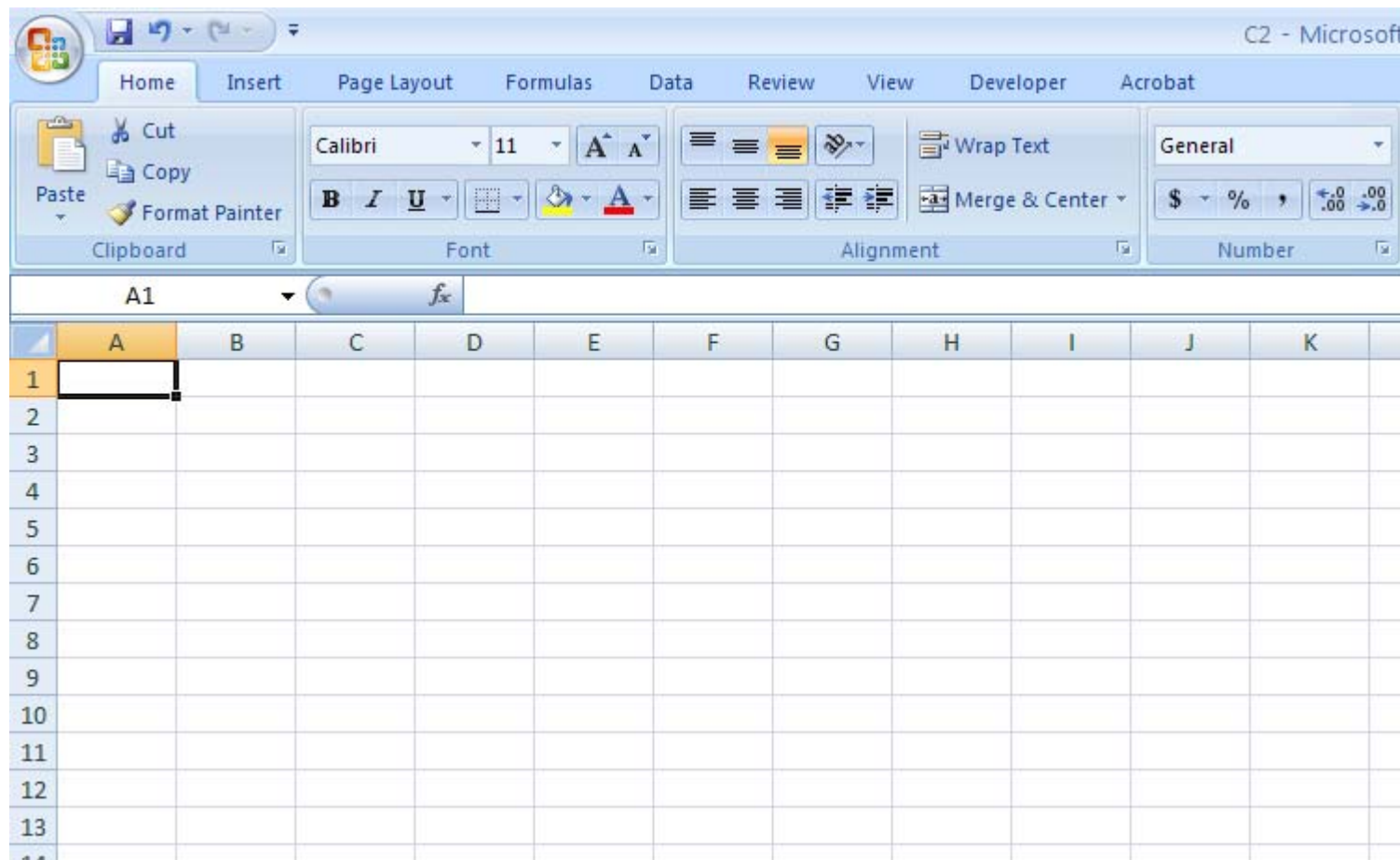
$$\left[\begin{array}{ccc|c} 4 & -2 & -5 & 11 \\ 1 & 1 & 1 & 2 \\ -2 & 3 & -2 & -14 \end{array} \right]$$

To put a matrix in reduced row echelon form in Excel, we carry out the row operations using the unique capabilities of Excel.

Reduced Row Echelon Form in Excel

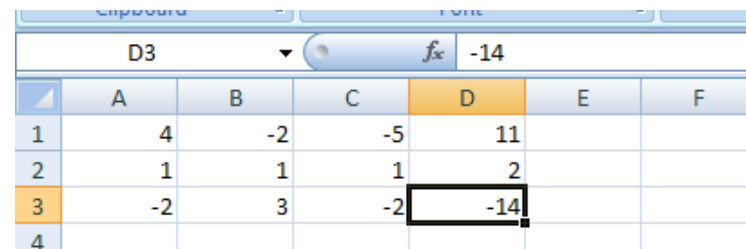
Put in the Original Matrix

1. Start Excel. Excel is a spreadsheet program that contains a huge table with many unique capabilities. The table is labeled by column and row with letters and numbers.



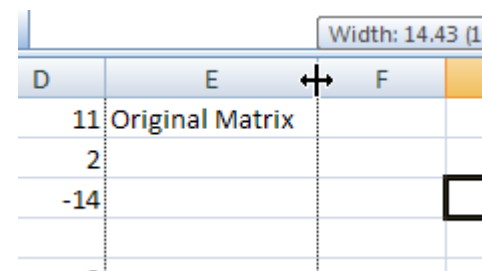
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- The cell called A1 is the cell in the column labeled A and the row labeled 1. Click your cursor in that cell and type a 4, the first entry in the augmented matrix for the system.
- The cell beneath A1 is labeled as A2 since it is in the A column and second row. Click your cursor in that cell and type a 1. Continue typing in the entries in the augmented matrix until they appear like those pictured to the right.



	A	B	C	D	E	F
1	4	-2	-5	11		
2	1	1	1	2		
3	-2	3	-2	-14		
4						


- In the cell labeled E1, type the words Original Matrix. Just above this entry, click on the column border. While holding the right mouse button, stretch it to make it wider.



D	E	F
11	Original Matrix	
2		
-14		
-		

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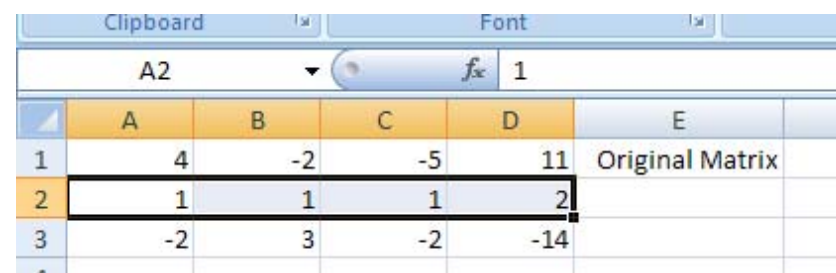
5. Click in cell E1 and then the Align Text

Right button, , from the Alignment Panel. This will move the text to the right side of the cell. As we proceed through the steps below, we'll use a similar style to document the row operations for each step.

D	E	F
11	Original Matrix	
2		
-14		

6. Click in cell A2. While holding the left mouse, drag the mouse to cell D2 so that cells A2 through D2 are selected.

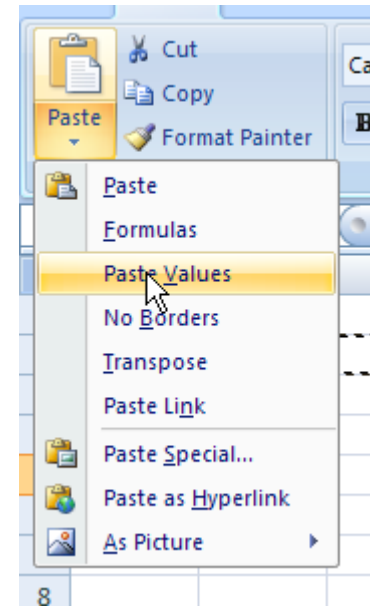
7. From the Clipboard Panel, select Copy. This copies the second row into the clipboard.



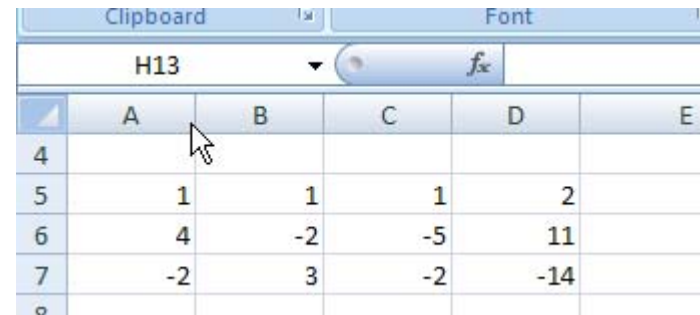
Clipboard		Font				
A2		<i>fx</i>	1			
	A	B	C	D	E	F
1	4	-2	-5	11	Original Matrix	
2	1	1	1	2		
3	-2	3	-2	-14		

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8. To paste this row into the first row of a new matrix, click in cell A5.
9. From the Clipboard Panel, locate the Paste button. Click on the down arrow under the word Paste. From the menu that appears, click on Paste Values. This will paste the second row of the original matrix into cells A5 through D5.



10. Repeat steps 6 through 9, but paste the first row in place of the second row.
11. Repeat steps 6 through 9 and paste the third row into the new matrix as the third row. The new matrix should look like the 1 to the right.

A screenshot of an Excel spreadsheet showing a 4x5 matrix. The matrix is located in cells A5 through D8. The values are: Row 5: 1, 1, 1, 2; Row 6: 4, -2, -5, 11; Row 7: -2, 3, -2, -14. The spreadsheet interface shows the 'Clipboard' and 'Font' tabs at the top, and the formula bar shows 'H13'. A mouse cursor is pointing at cell A5.

	A	B	C	D	E
4					
5	1	1	1	2	
6	4	-2	-5	11	
7	-2	3	-2	-14	
8					

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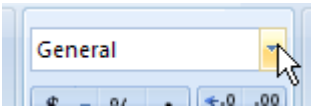
12. Now that there is a 1 in the first row, first column, we must use row operations to put 0's below it. The first row will not change in the new matrix so select cells A5 through D5 by drag clicking these cells. Press Copy from the Clipboard Panel.

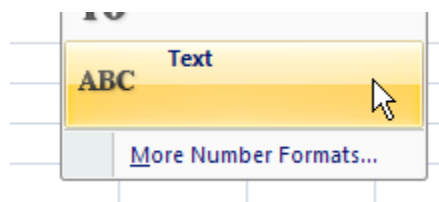
13. Click your mouse in cell A9. From the Clipboard Panel, locate the Paste button. Click on the down arrow under the word Paste. From the menu that appears, click on Paste Values. This pastes the first row into the new matrix unchanged.

	A	B	C	D	E
5	1	1	1	2	
6	4	-2	-5	11	
7	-2	3	-2	-14	
8					
9	1	1	1	2	
10					
11					
12					

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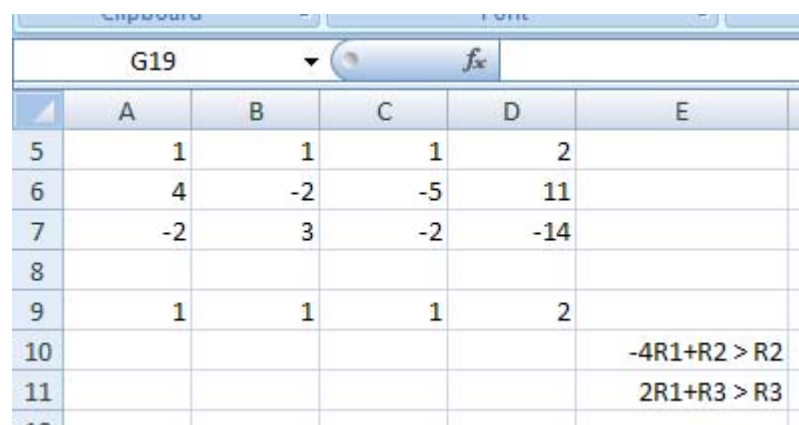
14. Document the row operations needed to put 0's below the leading 1. Click the mouse in cell E10. On the Number Panel, click on the handle next to the word

General, . From the menu that appears, select Text,



. Also click the right alignment button from the Alignment Panel. Now type the row operation $-4R1+R2 > R2$.

15. Click in cell E11 and repeat this process. In this case use the row operation $2R1+R3 > R3$.

A screenshot of an Excel spreadsheet. The active cell is G19. The spreadsheet shows a matrix in columns A through D and row operations in column E. Row 5 has values 1, 1, 1, 2. Row 6 has values 4, -2, -5, 11. Row 7 has values -2, 3, -2, -14. Row 8 is empty. Row 9 has values 1, 1, 1, 2. Row 10 has the operation $-4R1+R2 > R2$. Row 11 has the operation $2R1+R3 > R3$.

	A	B	C	D	E
5	1	1	1	2	
6	4	-2	-5	11	
7	-2	3	-2	-14	
8					
9	1	1	1	2	
10					$-4R1+R2 > R2$
11					$2R1+R3 > R3$

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16. To carry out the row operation $-4R_1 + R_2 > R_2$, click in cell A10 and type $= -\$A\$6 * A5 + A6$ and Enter. You should see the row operation applied to the first entry of the second row resulting in a 0. The \$ in this formula is an absolute reference that ensures that the opposite of the entry in A6 is always used to do the row operation.

17. Click the cursor in cell A10. Use your mouse to click on the handle in the lower right hand corner of the black box around the cell. While holding the left mouse button, drag the cursor to cell D10 and release the mouse button. This will fill the cells with the row operation.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E
5	1	1	1	2	
6	4	-2	-5	11	
7	-2	3	-2	-14	
8					
9	1	1	1	2	
10	0				$-4R_1 + R_2 > R_2$
11					$2R_1 + R_3 > R_3$
12					

The formula bar for cell A10 shows: $= -\$A\$6 * A5 + A6$. A black box with a handle is around cell A10, and a plus sign is visible in cell D10, indicating the start of a fill operation.

The screenshot shows the same Excel spreadsheet after the fill operation. The formula bar for cell A10 remains $= -\$A\$6 * A5 + A6$.

	A	B	C	D	E
5	1	1	1	2	
6	4	-2	-5	11	
7	-2	3	-2	-14	
8					
9	1	1	1	2	
10	0	-6	-9	3	$-4R_1 + R_2 > R_2$
11					$2R_1 + R_3 > R_3$
12					

The values in cells A10, B10, C10, and D10 are now 0, -6, -9, and 3 respectively, representing the result of the row operation $-4R_1 + R_2$ applied to the first row of the matrix.

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18. Repeat this process in cell A11 with the formula $=-A\$7*A5+A7$. After filling the row, the first column of the matrix is in the proper format.

	A	B	C	D	E
5	1	1	1	2	
6	4	-2	-5	11	
7	-2	3	-2	-14	
8					
9	1	1	1	2	
10	0	-6	-9	3	-4R1+R2 > R2
11	0	5	0	-10	2R1+R3 > R3
12					

19. Select and copy the numbers in cells A9 through D9. Paste these numbers by clicking on cell A13 and selecting Paste Values.

20. Select and copy the numbers in cells A11 through D11. Paste these numbers by clicking on cell A15 and selecting Paste Values.

21. To put a 1 in the second row, second column, we need to carry out the row

	A	B	C	D	E
9	1	1	1	2	
10	0	-6	-9	3	-4R1+R2 > R2
11	0	5	0	-10	2R1+R3 > R3
12					
13	1	1	1	2	
14					=-1/6R2 > R2
15	0	5	0	-10	
16					

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operation $-1/6R_2 > R_2$. Put this label in cell E14.

22. To carry out the row operation $-1/6R_2 > R_2$, click in cell B14 and type $=1/6*B10$ and Enter.

23. Click in cell B14 again and drag the fill handle to D14 to apply the row operation to the columns to the right. Then click in cell B14 and drag the fill handle to A14.

	A	B	C	D	E
9	1	1	1	2	
10	0	-6	-9	3	$-4R_1+R_2 > R_2$
11	0	5	0	-10	$2R_1+R_3 > R_3$
12					
13	1	1	1	2	
14	0	1	1.5	-0.5	$=-1/6R_2 > R_2$
15	0	5	0	-10	
16					

24. Now we'll put 0's above and below the 1 we just created. Start by selecting the cells A14 through D14. Press Copy on the Clipboard Panel.

25. Click the mouse in cell A18 and select Paste Values from the Clipboard Panel.

	A	B	C	D	E
13	1	1	1	2	
14	0	1	1.5	-0.5	$=-1/6R_2 > R_2$
15	0	5	0	-10	
16					
17					
18	0	1	1.5	-0.5	
19					
20					

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26. In cells E17 and E19, label the row operations, like in step 14, needed to place 0's above and below the 1 just created in the second row

H22					
	A	B	C	D	E
13	1	1	1	2	
14	0	1	1.5	-0.5	$=-1/6R2 > R2$
15	0	5	0	-10	
16					
17					$-1R2+R1 > R1$
18	0	1	1.5	-0.5	
19					$-5R2+R3 > R3$
20					

27. Use row operations to put a 0 above the 1 in the second column.

B17					
	A	B	C	D	E
13	1	1	1	2	
14	0	1	1.5	-0.5	$=-1/6R2 > R2$
15	0	5	0	-10	
16					
17		0			$-1R2+R1 > R1$
18	0	1	1.5	-0.5	
19					$-5R2+R3 > R3$
20					

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28. Once the 1 has been created, click on cell B17 and use the fill handles to fill the rest of the first row.

H28		fx				
	A	B	C	D	E	
13	1	1	1	2		
14	0	1	1.5	-0.5	$=-1/6R2 > R2$	
15	0	5	0	-10		
16						
17	1	0	-0.5	2.5	$-1R2+R1 > R1$	
18	0	1	1.5	-0.5		
19					$-5R2+R3 > R3$	
20						

29. Use row operations and the fill handle to place a 0 under the 1 in the second column.

B19		fx $=-B\$15*B14+B15$				
	A	B	C	D	E	
17	1	0	-0.5	2.5	$-1R2+R1 > R1$	
18	0	1	1.5	-0.5		
19	0	0	-7.5	-7.5	$-5R2+R3 > R3$	
20						

30. Use row operations and the fill handle to put a 1 in the third row, third column.

C23		fx $=(1/\$C\$19)*C19$				
	A	B	C	D	E	
20						
21	1	0	-0.5	2.5		
22	0	1	1.5	-0.5		
23	0	0	1	1	$1/-7.5R3 > R3$	
24						

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31. Use row operations and the fill handle to put 0's in the rest of the third column.

C26		fx = -\$C\$22*C23+C22				
	A	B	C	D	E	
25	1	0	0	3	0.5R3 +R1 > R1	
26	0	1	0	-2	-1.5R3+R2 > R2	
27	0	0	1	1		
28						

C25		fx = -\$C\$21*C23+C21				
	A	B	C	D	E	
25	1	0	0	3	0.5R3 +R1 > R1	
26	0	1	0	-2	-1.5R3+R2 > R2	
27	0	0	1	1		
28						

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The matrix in the Excel worksheet,

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

corresponds to the system of equations

$$x = 3$$

$$y = -2$$

$$z = 1$$

Since we used row operations to go from the original system of equations to this system, it is equivalent to the original system of equations. This means that the solutions above are also solutions to the original system of equations.