Find the second derivative of $f(x)=\dfrac{x-1}{x+2}$.

Find the first derivative: Since this function is a quotient, we'll need to apply the quotient rule with $u=x-1$ and $v=x+2$. Taking the derivatives of these pieces yields

$$\begin{matrix} u=x-1 \\ v=x+2 \end{matrix} \rightarrow \begin{matrix} u'=1 \\ v'=1 \end{matrix}$$

Put these pieces into the quotient rule give us

$$f'(x)=\frac{(x+2)\cdot 1-(x-1)\cdot 1}{(x+2)^2}$$

Simplify the first derivative: Before we take another derivative, we should simplify the first derivative as much as possible. This will make the second derivative a lot easier. In this case, let's remove the parentheses and distribute the subtraction to give

$$\frac{(x+2)\cdot 1-(x-1)\cdot 1}{(x+2)^2}=\frac{3}{(x+2)^2}$$

Rewrite the first derivative: We could use the quotient rule again, but it is easier to rewrite the first derivative as

$$f'(x)=3(x+2)^{-2}$$

Take the second derivative: We'll need the chain rule to take this derivative with

$$\begin{matrix} \text{inside}=x+2 \\ \text{outside}=3x^{-2} \end{matrix} \rightarrow \begin{matrix} \text{inside}'=1 \\ \text{outside}'=-6x^{-3} \end{matrix}$$

Applying the chain rule yields

$$f''(x)=-6(x+2)^{-3}\cdot 1$$

We can rewrite this as

$$f''(x)=\frac{-6}{(x+2)^3}$$