For the function $f(x) = x^2 - 2x + 3$, answer each of the questions below.

a. Find the equation of the secant line passing through x = 1 and x = 5.

b. Find the equation of the tangent line at x = 1.

For each of these parts, we'll find the equation of a line using $y = mx + b$. For part a, we'll find the slope of the line using two points on the function. For part b, we'll find the slope of the line using the derivative of $f(x)$ at $x = 1$ or $f'(1)$.

a. The line must pass through the point (1, 2) and (5,18) since

$$f(1) = 1^2 - 2(1) + 3 = 2$$
$$f(5) = 5^2 - 2(5) + 3 = 18$$

The slope of the line is

$$m_{\sec} = \frac{18-2}{5-1} = 4$$

This gives us the secant line equation $y = 4x + b$. To find the value of b, substitute one of the points for x and y:

$$2 = 4(1) + b$$
$$-2 = b$$

The equation of the secant line is $y = 4x - 2$.

b. The tangent line must pass through (1, 2) with a slope given by $f'(1)$. The derivative is found by applying the definition of the derivative at a point,

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

We have already found that $f(1) = 2$ and the other function value is

$$
\begin{aligned}
f(1+h) &= (1+h)^2 - 2(1+h) + 3 \\
&= 1 + 2h + h^2 - 2 - 2h + 3 \\
&= h^2 + 2
\end{aligned}
$$

The definition of the derivative gives

$$
\begin{aligned}
f'(1) &= \lim_{h \to 0} \frac{h^2 + 2 - 2}{h} \\
&= \lim_{h \to 0} \frac{h^2}{h} \\
&= \lim_{h \to 0} h \\
&= 0
\end{aligned}
$$

The equation of the tangent line is $y = 0x + b$. Substitute (1, 2) to give

$$
\begin{aligned}
2 &= 0(1) + b \\
2 &= b
\end{aligned}
$$

The equation of the tangent line is $y = 2$.

The lines and function are graphed below.