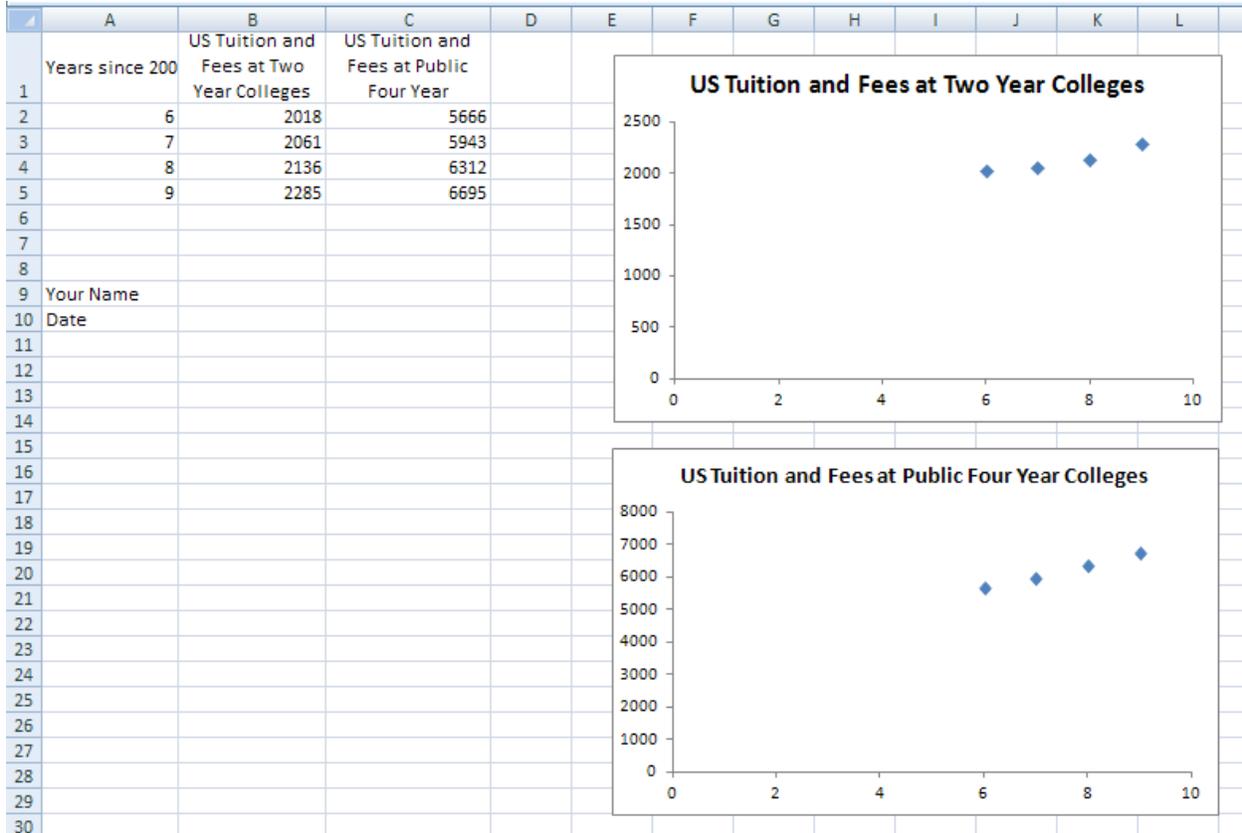


Tech Assignment: Find an Exponential Function for College Costs

In this technology assignment, you will find an exponential function that passes through two of the points on each of the scatter plots you created in an earlier technology assignment. You will need the file you created that looks like the one below. Your file will look slightly different since you are working with a different dataset.



Before we can put the exponential function that passes through two points on the scatter plot, we need to find the function by doing some mathematics. I'll show you how to do this math using the data for the US Tuition and Fees at two-year colleges. You should complete the same process for both sets of data for the state you are assigned.

Select the Points You'll Use

For this project, we'll find an exponential function that passes close to the data for tuition and fees at two-year colleges and four-year colleges by making the exponential function that passes through two of the data points. There are many possibilities we could choose. We could pass through the first two points, the first and third, second and third, the last two, or many other combinations of points.

Look at your data points. Visualize an exponential function passing through any two points. Pick two of the points to create a function that appears to represent the data best. These are the points you'll use to find an exponential function for your scatter plot.

Find the Equation of the Exponential Function

I will demonstrate how to find an exponential function through a pair of points using the ordered pairs $(6, 2018)$ and $(9, 2285)$. With these ordered pairs, the t values represent the number of years since 2000 and the y values represent the tuition and fees at US two-year colleges.

The equation of an exponential function is $f(t) = Pe^{rt}$. Since we are working with coordinates, we replace $f(t)$ with y ,

$$y = Pe^{rt}$$

Substitute each of the ordered pairs we want the exponential function to go through into this equation to give two equations,

$$2018 = Pe^{r \cdot 6}$$

$$2285 = Pe^{r \cdot 9}$$

If we can solve this system of equations for P and r , we will have the constants in an exponential function that passes through two of the points. Solve the first equation for P to give

$$P = \frac{2018}{e^{6r}}$$

Substitute the right side in place of P in the second equation to give

$$2285 = \frac{2018}{e^{6r}} e^{9r}$$

The exponential parts may be combined by subtracting the exponents,

$$2285 = 2018 e^{3r}$$

Now isolate the exponential piece by dividing both sides by 2018. This leaves us with

$$\frac{2285}{2018} = e^{3r}$$

We need to leave the left side as a fraction to avoid any rounding. Convert this equation to logarithmic form to give

$$\ln\left(\frac{2285}{2018}\right) = 3r$$

To get r by itself, multiply both sides by $\frac{1}{3}$,

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$$r = \frac{1}{3} \ln\left(\frac{2285}{2018}\right)$$

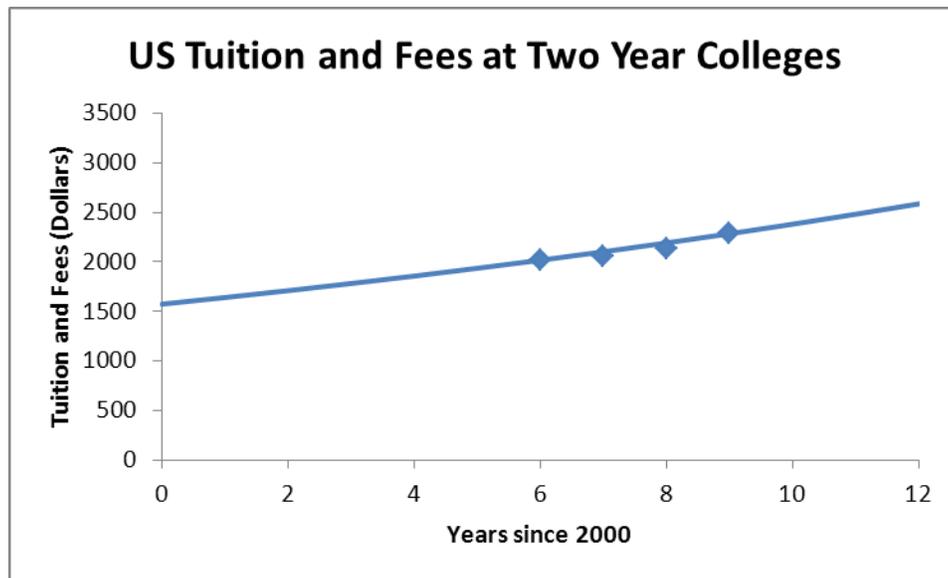
To get P , we must substitute this value into $P = \frac{2018}{e^{6r}}$. We avoid rounding and get

$$P = \frac{2018}{e^{6 \cdot \frac{1}{3} \ln\left(\frac{2285}{2018}\right)}}$$

The factor in front of the logarithm is equal to 2. Using the power rule for logarithms we can move it into the logarithm and simplify the denominator,

$$P = \frac{2018}{e^{2 \ln\left(\frac{2285}{2018}\right)}} = \frac{2018}{e^{\ln\left[\left(\frac{2285}{2018}\right)^2\right]}} = \frac{2018}{\left[\frac{2285}{2018}\right]^2}$$

Using the values for P and r from above gives us an exponential function that passes through the proper points.



Below I'll show you how to find this graph. Once you have completed the process for your first scatter plot, you will need to find another exponential function for the scatter plot for four-year colleges. This means you'll carry out the process above for two of the data points for four-year colleges in your state.

Before we start creating this graph, locate the earlier technology assignment where you created the scatter plots of two-year and four-year colleges. Copy this worksheet into a new workbook and name it something like p2_yourlastname.

Graph the Exponential Function on a Scatter Plot

1. To graph the exponential function we found above, we must create a table of values for the function. Start by clicking on cell A12 and typing 0. Then click on the cell underneath it, A13, and type a 1. This is the start of the x values we'll use for the line. We will start at 0 and graph points at increments of 1.

11	
12	0
13	1
14	
15	

2. To help us generate the x values, click on the cell A12. While holding the left mouse button down, drag the cursor to A13. This will select A12 and A13. Notice the dark black line surrounding the selection. In the lower right corner is a black box called the fill handle.

11	
12	0
13	1
14	

3. Click on the fill handle. The cursor will change to a cross. While holding down the left mouse button down, drag the cursor to cell A22. Use your mouse to grab the fill handle. While holding the left mouse button down, drag the mouse to cell A22. Release the mouse button to fill the rest of the table with y values.

11	
12	0
13	1
14	
15	
16	
17	
18	
19	
20	
21	
22	10
23	
24	

As you drag the cursor, notice the small box that appears. This tells us what entries will be filled in the cell. When you release the mouse button at cell A22, those numbers will be filled in cells A14 through A22. The numbers filled in follow the pattern established in cells A12 and A13.

This leaves us with the numbers 0 through 10 in cells A12 through A22. If we had started with 0 and 2 in A12 and A13, the column would be filled 0, 2, 4, 6,

4. In cells rows 7 and 8 we will put the values of r and P for the exponential function. In cell A7, type the letter r . In cell A8, type the letter P .

6			
7	r		
8	P		
9			

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- In cells B7 and B8 we'll put the values for r and P for two year colleges. In cell B7, type the formula for r you see below with appropriate changes for your data.

6		
7	r	=LN(2285/2018)/3
8	P	

In cell B8, type the formula for P you see below. You will have different numbers since your data is different from the data used to get this formula.

6		
7	r	0.041419701
8	P	=2018/(2285/2018)^2
9		

- In cell B12, we want to put the equation of the exponential function. This is done by clicking on cell B12 and type

11		
12	0	=B\$8*EXP(\$B\$7*A12)
13	1	
14	2	

Notice the \$ signs in the formula. When you need to fix the location of a number in your worksheet, you use absolute references. This type of reference uses a \$ before the row and column location in the cell reference. When we fill the rest of this column with the formula for the exponential function, the values for r and P will be fixed for the formula. However the corresponding x values will change in each row.

- Press **Enter** on the keyboard to calculate the y value.

11		
12	0	1573.950525
13	1	
14	2	

- To compute the rest of the y values, we'll use a fill. Click on cell B12 to select it.

11		
12	0	1573.950525
13	1	
14	2	

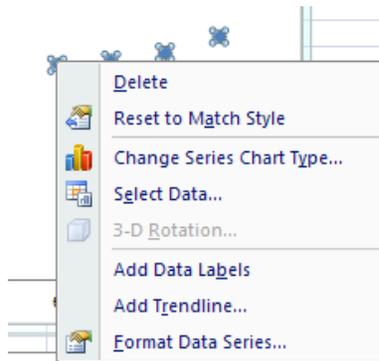
- Use your mouse to grab the fill handle. While holding the left mouse button down, drag the mouse to cell A22. Release the mouse button to fill the rest of the table with y values.

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11			11	
12	0	1573.950525	12	1573.950525
13	1		13	1640.512049
14	2		14	1709.888423
15	3		15	1782.198687
16	4		16	1857.566913
17	5		17	1936.122422
18	6		18	2018
19	7		19	2103.340137
20	8		20	2192.289263
21	9		21	2285
22	10		22	2381.631424
23	11		23	2482.349339
24	12		24	2587.326561
25			25	

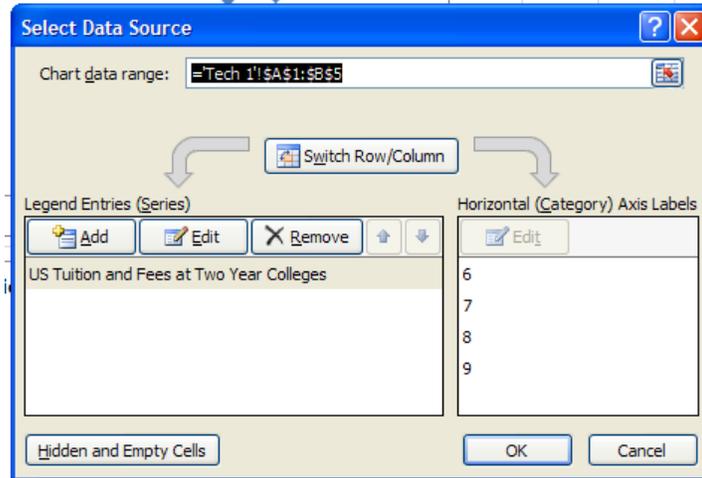
These are the values we'll use to make an exponential function on the scatter plot. Did you notice that the two values at $x = 6, 9$ match the desired data values? This is good because it means the exponential function passes through the points we want it to pass through.

- To add the exponential function to the scatter plot, right mouse click on one of the data points in the scatter plot. A menu will appear like the one to the right. Choose "Select Data" from this menu.

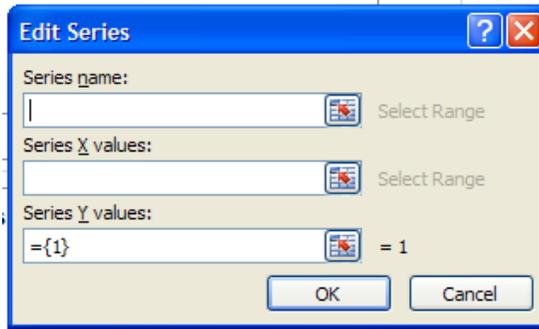


- The Select Data Source box will appear. Under Legend Entries (Series), you'll see the title of the existing scatter plot. We want to add another dataset to this scatter plot. Click on **Add**.

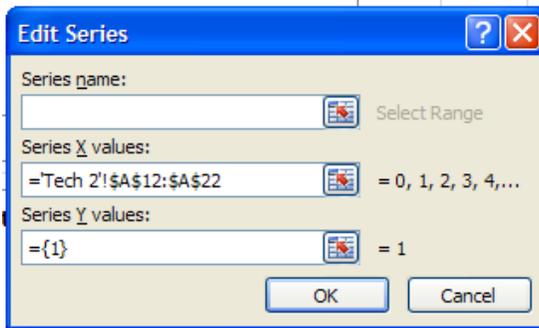
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12. To put the function on the scatter plot, we need to enter the location of the exponential function's points in the Edit Series box.



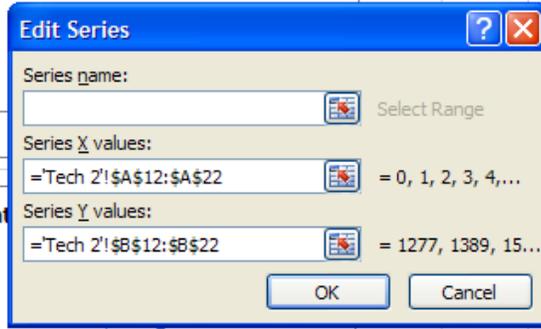
13. Click in the box under "Series X values" to place the insertion point in that box. Now click on cell A12 and drag select (hold the left mouse button) to cell A22. If your x values are not in cells A12 through A22, you may need to drag select a slightly different set of cells.



The location of the cells is pasted into "Series X values".

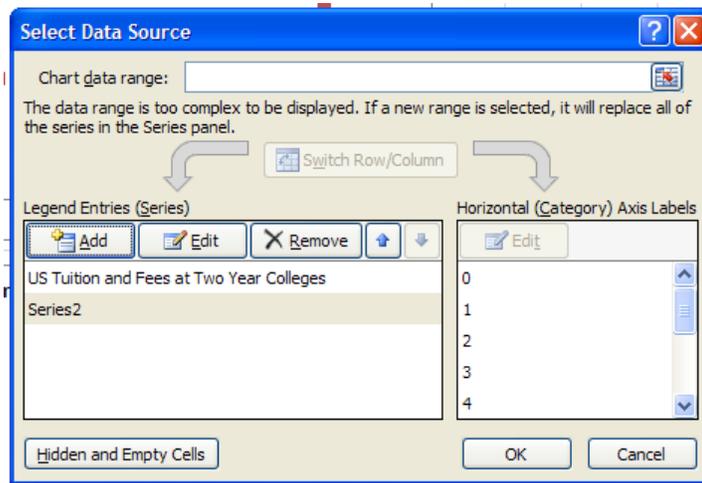
14. Put the insertion point in the box under "Series Y values". Delete any entry in that box. Now click on cell B12 and drag select (hold the left mouse button) to cell B22. If your x values are not in cells B12 through B22, you may need to drag select a slightly different set of cells.

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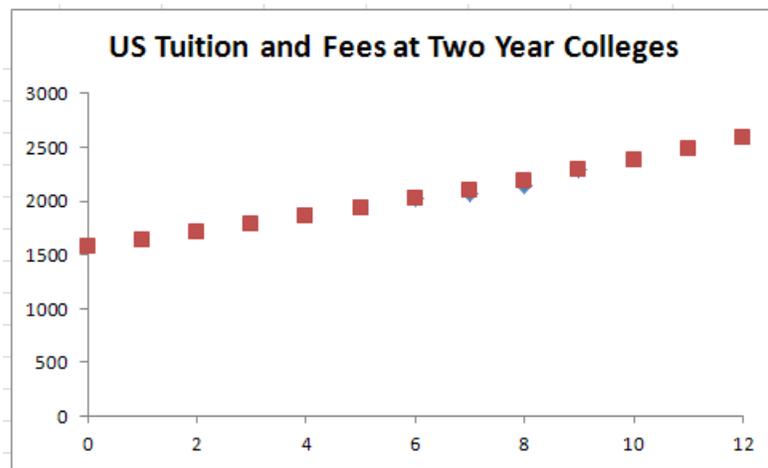
The location of the selected cells is pasted into "Series Y values". Click **OK**.

15. You'll return to the "Select Data Source" box.

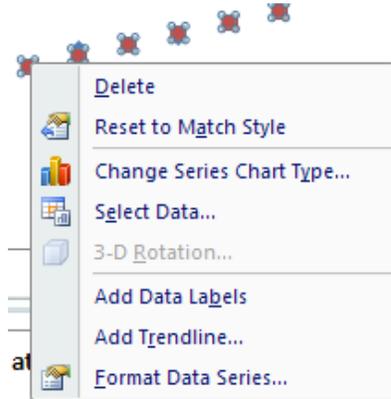


Click **OK** to see the new scatter plot.

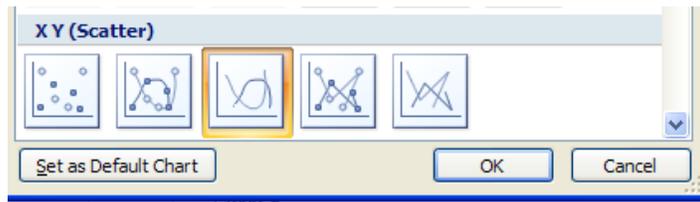
16. By default, the exponential function is graphed as a scatter plot and not as a line graph. We need to modify the red points so that we see a curve and not points on the exponential function.



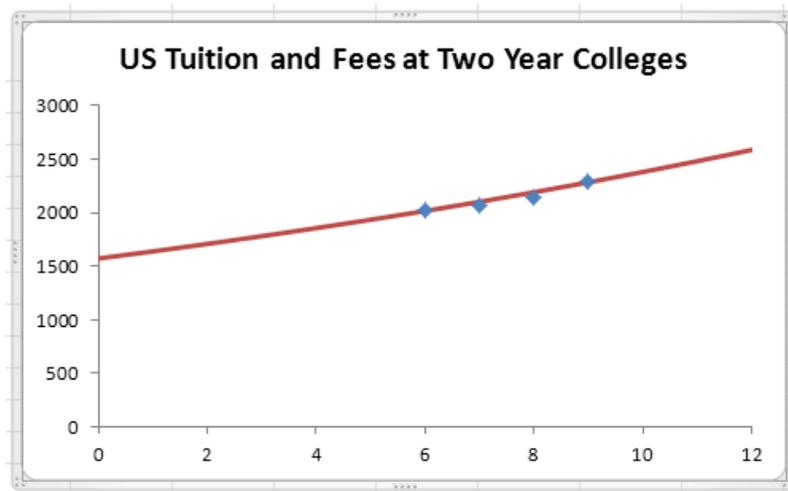
17. Right mouse click on one of the data points on the line. From the menu that appears, select “Change Series Chart Type...”.



18. Select the line graph from “XY (Scatter)”. Click **OK**.



The points will change to a curve on the scatter plot.



19. To finish the graph, we will add labels on each axis to describe what is being graphed. To begin doing this, click on the outside edge of the graph’s window to select it. You’ll see a double lined edge appear around the window. Along the top of Excel, you should see a series of tabs entitled “Chart Tools”. Click on the “Layout” tab.

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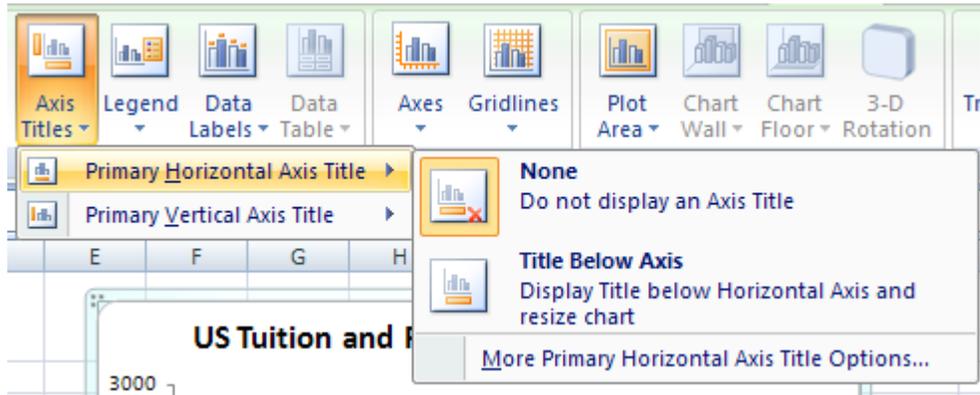


If you are using Excel 2013, click on the + on the right side to add axes titles. Then jump to step 22 to edit the axes titles.

20. Locate the “Labels” panel. Select “Axis Titles”.



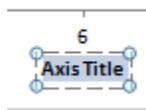
21. Now select “Primary Horizontal Axis Title” and “Title Below Axis”.



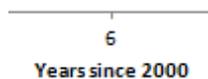
22. You’ll see a label appear under the horizontal axis.



23. Since “Axis Title” is not very descriptive, select the text (or delete it).

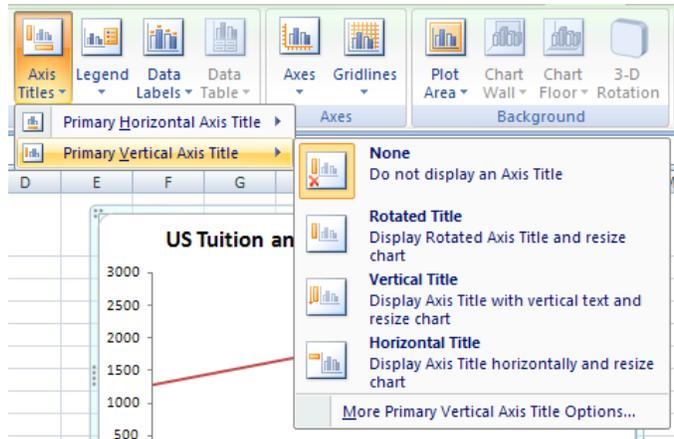


24. Change the text in the label to “Years since 2000”.

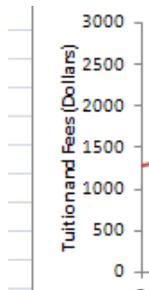


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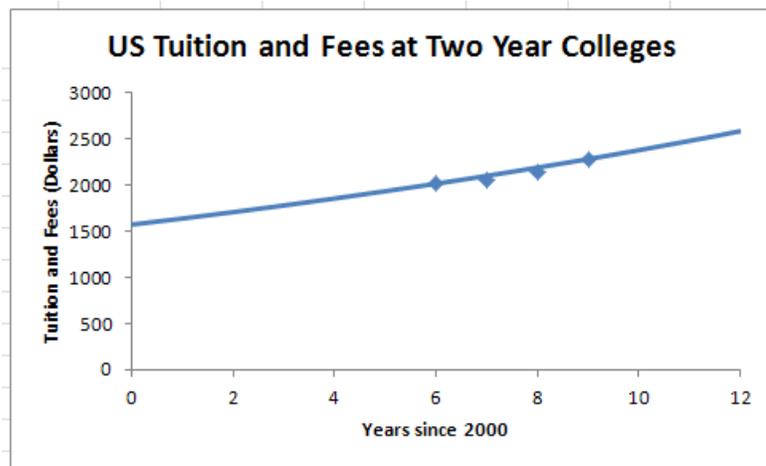
25. Let's follow the same process to add a label to the vertical axis. Make sure the graph is selected and choose "Axis Titles" from the Labels panel. In this case, select "Primary Vertical Axis Title" and "Rotated Title".



26. Edit the label that appears appropriately. Make sure you include the units, dollars, on the axis title.



27. The new graph is shown below.



A good graph includes a title, horizontal axis title, vertical axis title, and appropriate scatter plots and/or line graphs.

28. To complete this assignment, repeat the steps you just completed to find an exponential function for the scatter plot containing the four-year college data. Once you have an exponential function on your scatter plot for four-year colleges, save your Excel file. This is the file you will submit (using the data from the state you are assigned) for this technology assignment.