

The U-Drive Rent-A-Truck company plans to spend \$7 million on 200 new vehicles. Each commercial van will cost \$35,000, each small truck \$30,000, and each large truck \$50,000. Past experience shows that they need twice as many vans as small trucks. How many of each vehicle can they buy?

Start by defining the variables:

V : number of commercial vans to buy

S : number of small trucks to buy

L : number of large trucks to buy

Now let's look at the key information and the corresponding equation:

buy 200 new vehicles $\rightarrow V + S + L = 200$

spend 7 million $\rightarrow 35000V + 30000S + 50000L = 7000000$

need twice as many vans as small trucks $\rightarrow V = 2S$

So the system we need to solve is

$$\begin{aligned} V + S + L &= 200 \\ 35000V + 30000S + 50000L &= 7000000 \\ V &= 2S \end{aligned}$$

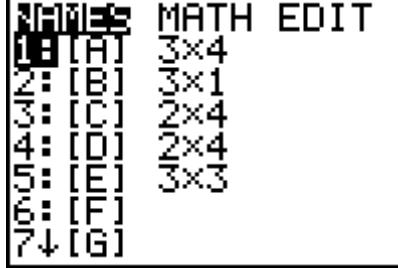
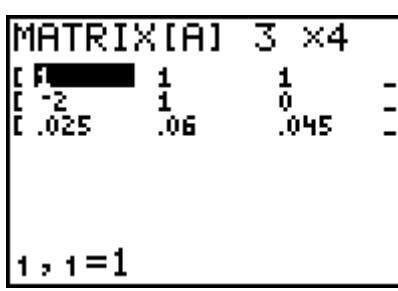
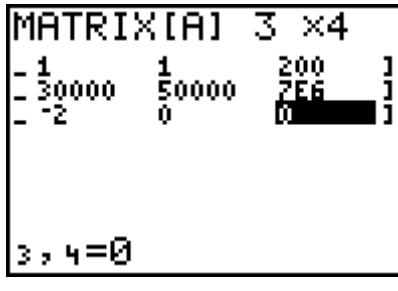
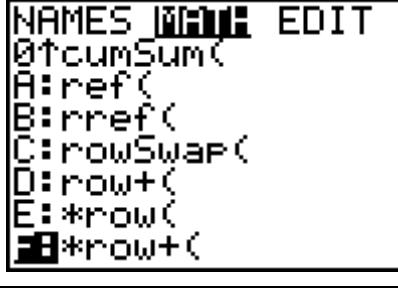
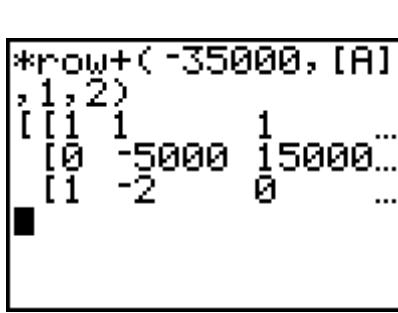
Rewriting in the proper form, we get

$$\begin{aligned} V + S + L &= 200 \\ 35000V + 30000S + 50000L &= 7000000 \\ V - 2S &= 0 \end{aligned}$$

The augmented matrix for this system is

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 200 \\ 35000 & 30000 & 50000 & 7000000 \\ 1 & -2 & 0 & 0 \end{array} \right]$$

To put this in row echelon form, follow the steps below.

<ol style="list-style-type: none"> Start by pressing [2nd][x^-1] to open the matrix menu. From this menu you can access the NAMES of different matrices, do MATH with matrices and EDIT matrices. Use [▶] to move to EDIT. 	 <p>NAMES MATH EDIT 1: [A] 3×4 2: [B] 3×1 3: [C] 2×4 4: [D] 2×4 5: [E] 3×3 6: [F] 7↓[G]</p>
<ol style="list-style-type: none"> Press 1 or highlight 1: [A] and press [ENTER]. Change the matrix to being a 3 by 4 matrix. To do this, simply type that size in place of the existing size. Use your cursor control buttons to move to the different entries in the matrix. Change them to the augmented matrix we found earlier. 	 <p>MATRIX[A] 3 ×4 $\begin{bmatrix} 1 & 1 & - & - \\ -2 & 1 & 0 & - \\ .025 & .06 & .045 & - \end{bmatrix}$ $1, 1=1$</p>
<ol style="list-style-type: none"> You can press [ENTER] to move you through the matrix or use the cursor control. Once you have entered the matrix, press [2nd][MODE] to quit the matrix editor and to return to the home screen. You may need to press [CLEAR] to clean up the home screen. 	 <p>MATRIX[A] 3 ×4 $\begin{bmatrix} 1 & 1 & 200 &] \\ -35000 & 50000 & 250 &] \\ -2 & 0 & 0 &] \end{bmatrix}$ $3, 4=0$</p>
<ol style="list-style-type: none"> Press [2nd][x^-1] to enter the matrix menu. Move the cursor to the MATH menu. Scroll down to F: *row+(). 	 <p>NAMES MATH EDIT $\text{gtcumSum}()$ $A:\text{ref}()$ $B:\text{rref}()$ $C:\text{rowSwap}()$ $D:\text{row}()$ $E:\text{*row}()$ F: *row+()</p>
<ol style="list-style-type: none"> Press [ENTER] to paste the command to the home screen. We want to carry out the row operation $-35000 R1 + R2 \rightarrow R2$. To do this type in -35000 followed by [,]. To let the calculator know that you are working with the matrix named A, press [2nd][x^-1]. Highlight the matrix A under NAMES and press [ENTER]. This will 	 <p>*row+(-35000, [A], 1, 2) $\begin{bmatrix} 1 & 1 & 1 & ... \\ 0 & -5000 & 15000 & ... \\ 1 & -2 & 0 & ... \end{bmatrix}$</p>

<p>paste the name of the matrix into the command on the homescreen.</p> <p>14. Now press $\boxed{,} \boxed{1} \boxed{,} \boxed{2} \boxed{)}$ to indicate that you'll be multiplying row 1 by -35000 and adding it to row 2 with the result being placed in row 2.</p> <p>15. Press ENTER to carry out the command. As desired, the first entry in the second row is zero.</p>	
<p>16. This is a key step. You need to save the matrix you just created. To store the matrix, press STO \blacktriangleright 2nd $[x^{-1}]$.</p> <p>17. To put in the name of the matrix you want to store, press scroll to the name you want to use with the $\boxed{\square}$ button. Often it is easiest to simply use A again. To use the name A you can just press ENTER. This will paste [A] to the home screen.</p> <p>18. Press ENTER to store the matrix.</p>	<pre>[[1 1 1 [0 -5000 15000... [1 -2 0 ... Ans→[A] [[1 1 1 [0 -5000 15000... [1 -2 0 ...]</pre>
<p>19. Repeat steps 10 through 18 to carry out the row operation $-1 R1 + R3 \rightarrow R3$. The command you'll use after carrying out steps 10 through 15 is $*row+(-1,[A], 1,3)$. Make sure you save the matrix by following steps 16 through 18.</p>	<pre>[[1 1 1 [0 -5000 15000... [0 -3 -1 ... Ans→[A] [[1 1 1 [0 -5000 15000... [0 -3 -1 ...]</pre>
<p>20. In this step, we need to change the -5000 to 1 in the second row by multiplying by $-\frac{1}{5000}$ or $-\frac{1}{5000} R2 \rightarrow R2$. Go to the MATRIX MATH menu by pressing 2nd $[x^{-1}] \blacktriangleright$.</p> <p>21. Scroll down to E: $*row($ and press ENTER. The command will be pasted to the homescreen.</p> <p>22. Complete the command by pressing $\boxed{(-} \boxed{1} \boxed{\div} \boxed{5} \boxed{0} \boxed{0} \boxed{0}, \boxed{2nd} \boxed{x^{-1}} \boxed{ENTER}, \boxed{,} \boxed{2} \boxed{)}$.</p>	<pre>NAMES MATH EDIT 0: cumSum(A: ref(B: rref(C: rowSwap(D: row+(E: *row(F: *row+(</pre>

23. Press **[ENTER]** to execute the command. Notice that the -5000 has changed to 1.

```
*row(-1/5000,[A])
,2)
[[1 1 1 200 ]
 [0 1 -3 0 ]
 [0 -3 -1 -200]]
■
```

24. Press **[STO] [2nd][x-1][ENTER][ENTER]** to store the matrix as A.

```
[[1 1 1 200 ]
 [0 1 -3 0 ]
 [0 -3 -1 -200]]
Ans→[A]
[[1 1 1 200 ]
 [0 1 -3 0 ]
 [0 -3 -1 -200]]
```

25. Repeat steps 10 through 18 to carry out the row operation $-3R_2 + R_3 \rightarrow R_3$. The command you'll use after carrying out steps 10 through 15 is ***row+(-3,[A],2,3)**. Make sure you save the matrix by following steps 16 through 18.

```
*row+(-3,[A],2,3)
[[1 1 1 200 ]
 [0 1 -3 0 ]
 [0 0 -10 -200]]
```

26. Repeat steps 20 through 23 to carry out the row operation $-\frac{1}{10}R_3 \rightarrow R_3$. The command should look like ***row(-1/10,[A],3)**.

```
*row(-1/10,[A],3)
[[1 1 1 200 ]
 [0 1 -3 0 ]
 [0 0 1 20 ]]
```

This matrix is equivalent to the system

$$V + S + L = 200$$

$$S - 3L = 0$$

$$L = 20$$

If we backsubstitute, we get

$$S - 3(20) = 0$$

$$S - 60 = 0$$

$$S = 60$$

and

$$V + 60 + 20 = 200$$

$$V + 80 = 200$$

$$V = 120$$

So they need to buy 20 large trucks, 60 small trucks, and 120 commercial vans.