Section 1.1 Functions

Question 1 - What are independent and dependent variables?

Question 2 - What is a linear function?

Question 3 - What is function notation?

Question 1 - What are independent and dependent variables?

Key Terms

Equation

Independent Variable   Dependent Variable

Summary

An equation relates the relationship between variables. If the equation has two variables, one may be designated the independent variable and the other is the dependent variable. The independent variable represents where input values are supplied in the equation. The dependent variable outputs values that depend upon what is put into the independent variable.

Typically, an equation is solved for the dependent variable to make it easier to calculate the output for some value of the independent variable. If this is not the case, then we need to specify what variable is the independent variable and which variable is the dependent variable.

Notes

<u>Guided Example</u>   <u>Practice</u>

| Suppose the demand Q and the price P are related by the demand equation

 $$5P + 20Q = 100$$

 a.   Write the equation so that Q is the independent variable.

 Solution If Q is the independent variable, the other variable, P, must be the dependent variable. To make P the dependent variable, solve for P:

 $5P = -20Q + 100$ Subtract 20Q from both sides
 $P = -4Q + 20$ Divide both sides by 5

 b.   Write the equation so that P is the independent variable.

 Solution To make P the independent variable, solve for Q:

 $20Q = -5P + 100$ Subtract 5P from both sides
 $Q = -\frac{1}{4}P + 5$ Divide both sides by 20 | 1. Suppose the demand Q and the price P are related by the demand equation

 $$12P + 6Q = 36$$

 a.   Write the equation so that Q is the independent variable.

 b.   Write the equation so that P is the independent variable. |

Question 2 - What is a linear function?

<u>Key Terms</u>

Equation   Linear Function

<u>Summary</u>

In Finite Mathematics, we typically work with equations that correspond to linear functions. A linear function in two variables has a graph that is a line. Any linear function in the variables x and y can be solved for y and put in the form

$$y = mx + b$$

This can be extended to other variables. For instance, supply and demand equations typically use the variables p and q. If q is the independent variable, a linear function could be written in the form

$$p = mq + b$$

This is simply the same format as before with x and y replaced by q and p.

<u>Notes</u>

Guided Example | Practice

Which of the function below are linear functions?

a. $5x + 3y = 9$

Solution To be a linear function, the equation must be able to be put in the form $y = mx + b$. Solve the equation y:

$3y = -5x + 9$ Subtract -5x from both sides

$y = -\frac{5}{3}x + 3$ Divide both sides by 3

Since this is in the proper format with $m = -\frac{5}{3}$ and $b = 3$.

b. $y = 2x^2 + 7$

Solution This equation is already solved for y. Because of the power on the variable, it is not possible to put in the form $y = mx + b$ so the equation does not represent a linear function.

c. $y = \dfrac{1}{x} + 10$

Solution This equation is also solved for y. Since it cannot be put in the form $y = mx + b$, the equation does not correspond to a linear function.

1.   Which of the function below are linear functions?

a. $2x - 7y = 14$

b. $y = 6x^2 - 10$

c. $y = \dfrac{2}{x^2} - 3$

<u>Guided Example</u>   <u>Practice</u>

The price per unit P (in dollars) is related to the quantity Q (in boxes) of a product demanded by consumers according to $$5P + 4Q = 200$$ Write the equation as a linear function of Q. **Solution** To write this equation so that it is a function of Q (P is the dependent variable), we need to solve for P: $$5P = -4Q + 200 \quad \text{Subtract } 4Q \text{ from both sides}$$ $$P = -\tfrac{4}{5}Q + 40 \quad \text{Divide both sides by 5}$$ This equation corresponds to a linear function of the form $P = mQ + b$.	2. The price per unit P is related to the quantity Q (in cases) of a product demanded by consumers according to $$2P + 3Q = 360$$ Write the equation as a linear function of P.

Question 3 - What is function notation?

<u>Key Terms</u>

Function Notation

<u>Summary</u>

Suppose you are given a linear function $y = 2x + 1$. To find the value of the dependent variable in this equation when x is 5, you might be asked to substitute $x = 5$ into the equation:

$$y = 2(5) + 1 = 11$$

This tells us two things. First, the value $x = 5$ corresponds to $y = 11$. Second, the ordered pair $(x, y) = (5, 11)$ on the graph of the linear function.

Function notation provides a more compact way of stating the relationship between x and y. In function notation, we replace the dependent variable with a name for the function. Next to the name, we place the name of the independent variable in parentheses. This helps us to distinguish variables from constants in the function. For the function above, we might name the function f and write

$$f(x) = 2x + 1$$

In this context, we say "f of x" when saying $f(x)$. If we want to find the value of the dependent variable y at $x = 5$ we simply write find $f(5)$. Then we calculate

$$f(5) = 2(5) + 1 = 11$$

We can also find the value of the independent variable x when $y = 23$ by saying find the value of x for which $f(x) = 23$. In this case we set the function's formula equal to 23 and solve for x:

$$2x + 1 = 23$$
$$2x = 22$$
$$x = 11$$

In this course, most applications are from business, economics, and business. You will see many names for functions such as D for a demand function, S for a supply function, C for a cost function, R for a revenue function and P or Pr for a profit function. It is very common for these functions to be written so that the independent variable is the quantity Q. In fact, you may even see function notation used to indicate the relationship between functions.

$$\text{Revenue} = \text{Price} \cdot \text{Quantity} \quad \Leftrightarrow \quad R(Q) = D(Q) \cdot Q$$

$$\text{Profit} = \text{Revenue - Cost} \quad \Leftrightarrow \quad P(Q) = R(Q) - C(Q)$$

<u>Notes</u>

Example	Practice
Suppose $f(x) = 2x + 7$.	1. Suppose $f(x) = 3x - 6$.
a. Find $f(5)$.	a. Find $f(10)$.
Solution The symbol $f(5)$ indicates that $x = 5$ should be substitute for x in the right side of the function, $$f(5) = 2(5) + 7 = 17$$ This tells us that the point (5, 17) is on the graph of this function.	
b. Find $f\left(\frac{1}{2}\right)$	b. Find $f\left(\frac{2}{3}\right)$
Solution The symbol $f\left(\frac{1}{2}\right)$ indicates that $x = \frac{1}{2}$ should be substitute for x in the right side of the function, $$f\left(\tfrac{1}{2}\right) = 2\left(\tfrac{1}{2}\right) + 7 = 8$$ This tells us that the point ($\frac{1}{2}$, 8) is on the graph of this function.	
	c. Find $f(z)$.
c. Find $f(t)$.	
Solution Instead of replacing x with a value, $f(t)$ indicates that we should replace x with t to give $$f(t) = 2t + 7$$	d. Find the value of x for which $f(x) = 30$.
d. Find the value of x for which $f(x) = 19$.	
Solution In previous parts, a value for the independent variable was supplied. In this part, the value of the dependent variable is supplied. Replace $f(x)$ with 19 and solve for x: $$19 = 2x + 7$$ $$12 = 2x$$ $$6 = x$$	

Guided Example | Practice

Suppose that the demand and price for a certain DVD is related by the following equation

$$p = D(q) = -1.5q + 60$$

where p is the price (in dollars) and q is the quantity demanded (in thousands).

a. Find the price when the demand is 5000.

   Solution Since the quantity demanded is in thousands, 5000 DVDs corresponds to $q = 5$ Substitute this value into the function gives

   $$p = D(5) = -1.5(5) + 60 = 52.5 \text{ dollars}$$

b. What quantity demanded corresponds to a price of $27?

   Solution Since the demand function outputs price, we need to set the function equal to 27 and solve for q:

   $-1.5q + 60 = 27$

   $\quad -1.5q = -33$ Subtract 60 from both sides

   $\quad\quad q = \dfrac{-33}{-1.5}$ Divide both sides by -1.5

   $\quad\quad q = 22$

   This corresponds to 22,000 DVDs.

2. Suppose that the demand and price for a certain GPS unit is related by the following equation

$$p = D(q) = -0.5q + 350$$

where p is the price (in dollars) and q is the quantity demanded (in hundreds).

a. Find the price when the demand is 1000.

b. What quantity demanded corresponds to a price of $325?

Section 1.2 Applications of Linear Functions

Question 1 - What are the pieces of a linear function?

Question 2 - How do you graph a linear function?

Question 3 - How do you find a linear function through two points?

Question 4 - How do you find cost and revenue functions?

Question 5 - What are demand and supply?

Question 1 - What are the pieces of a linear function?

Key Terms

Slope   Rate

Marginal

Summary

In $y = mx + b$, the constant m represents the slope of the line and tells us the rate at which the variables are changing with respect to each other. The unit on the slope is the units of the dependent variable divided by the units of the independent variable. In the context of cost, revenue, and profit functions, the slope is typically referred to as marginal cost, marginal revenue, and marginal profit. These quantities indicate how the cost / revenue / profit will change when production is increased by one unit.

The constant b represents the vertical intercept of the line and indicates the value of the dependent variable when the independent variable is zero.

Notes

Guided Example	Practice

Find the slope and vertical intercept of each line.
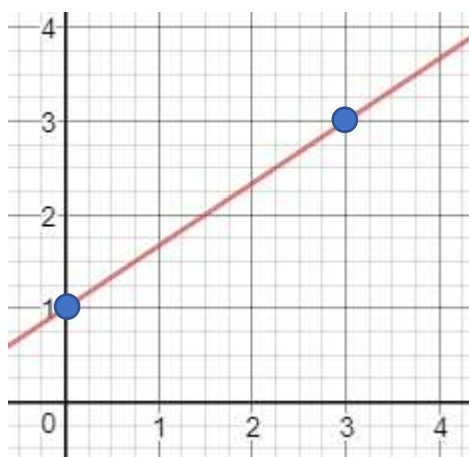
a. $y=-\frac{3}{4}x+10$

Solution The function is in the form $y = mx + b$ so $m=-\frac{3}{4}$ and $b=10$.

b. $2x + 4y = 10$

Solution Solve the equation for y to yield $y=-\frac{1}{2}x+\frac{5}{2}$. The slope is $m=-\frac{1}{2}$ and the vertical intercept is $b=\frac{5}{2}$

c.

Solution Locate two points on the line such as (0, 1) and (3, 3). The slope between these points is
$$m = \frac{3-1}{3-0} = \frac{2}{3}$$

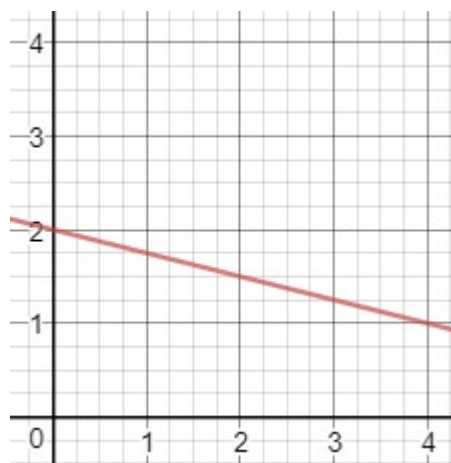Since the vertical intercept is 1, the equation of the line is
$$y=\frac{2}{3}x+1$$

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1. Find the slope and vertical intercept of each line.

a. $y=\frac{1}{3}x-4$

b. $3x + 6y = 18$

c.

Guided Example   Practice

Find an equation of the line having slope -3 and containing (3, -1).	2. Find an equation of the line having slope 2 and containing (5, 8).
Solution Start from $y = mx + b$ and substitute the value -3 for the slope to give $$y = -3x + b$$ To find the value of b, set $x = 3$ and **y** = -1 and solve for *b*: $$-1 = -3(3) + b$$ $$-1 = -9 + b$$ $$8 = b$$ With $m = -3$ and $b = 8$, the equation of the line is $$y = -3x + 8$$	

Guided Example   Practice

Find an equation in slope-intercept form for the line that passes through $(2, -6)$ with $m = 0$.	3. Find an equation in slope-intercept form for the line that passes through (5, 3) with $m = 0$.
Solution Start from $y = mx + b$ and substitute the value 0 for the slope to give $$y = 0x + b$$ To find the value of b, set $x = 2$ and **y** = -6 and solve for *b*: $$-6 = 0(2) + b$$ $$-6 = 0 + b$$ $$-6 = b$$ With $m = 0$ and $b = -6$, the equation of the line is $$y = -6$$	

<u>Guided Example</u>   <u>Practice</u>

The price per unit P (in dollars) is related to the quantity Q (in boxes) of a product demanded by consumers according to $$P=-\tfrac{4}{5}Q+40$$ a. What is the slope of the function in part a? **Solution** Since the equation matches $P = mQ + b$ with $m=-\tfrac{4}{5}$, the slope of the line is $-\tfrac{4}{5}$. b. Interpret the slope in part a. **Solution** For this problem, the dependent variable units are dollars and the independent variable units are boxes, the units on the slope are dollars per box. A slope of $$m=-\tfrac{4}{5}=-0.8\tfrac{\text{dollars}}{\text{box}}$$ means that lowering the unit price by 0.8 dollars will increase consumer demand by 1 unit. c. What is the vertical intercept of the function in part a? **Solution** Since the equation matches $P = mQ + b$ with $b = 40$, the vertical intercept is 40.	4. The price per unit P is related to the quantity Q (in cases) of a product demanded by consumers according to $$P=-\tfrac{3}{2}Q+180$$ a. What is the slope of the function in part a? b. Interpret the slope in part a. c. What is the vertical intercept of the function in part a?

**Question 2** -   How do you graph a linear function?

<u>Key Terms</u>

None

<u>Summary</u>

To graph a linear function, determine the slope of the line and a single point on the line. For functions in the form $y = mx + b$, the point is typically the vertical intercept. Then use the rise and run from the slope to locate a second point on the line.

For example, the slope of the line $y = \frac{1}{2}x + 2$ is $\frac{1}{2}$. This means that we can locate another point to units to the right and up 1 unit from the vertical intercept.
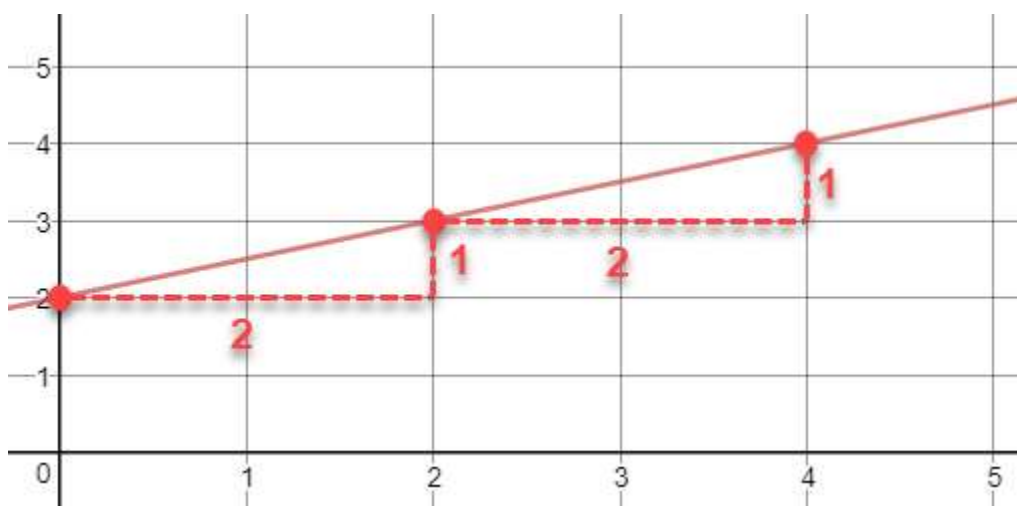
More points can be located in a similar matter. The graph is simply a line through these points.

For slopes that are negative, you must think of the negative as a part of either the numerator or the denominator. Depending on its location, you would move to the left or down instead of to the right or up.

<u>Notes</u>

Guided Example	Practice
Graph the function $f(x)=\frac{3}{2}x+3$.	1. Graph the function $f(x)=-\frac{1}{3}x+5$.
Solution Start by identifying the slope $m=\frac{3}{2}$ and the vertical intercept $b=3$. Label the vertical intercept on the graph at (0, 3).	
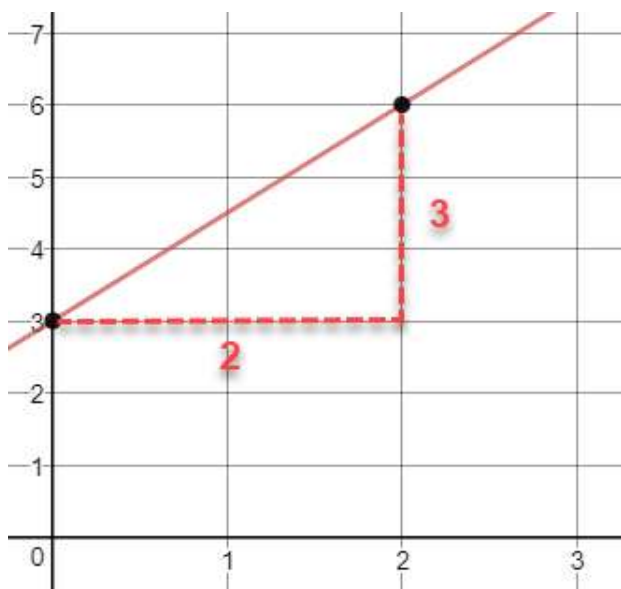
Graph:

To find another point on the line, use the fact that the slope is $m=\frac{3}{2}$. A second point on the line is (2, 6). Connect the two points to get the line pictured above.

Question 3 - How do you find a linear function through two points?

<u>Key Terms</u>

None

<u>Summary</u>

To find a linear function that passes through a pair of points, start by computing the slope of the line connecting the points. If (x_1, y_1) and (x_2, y_2) are the coordinates of the point, the slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Once the slope is calculated, substitute it into the linear function $y = mx + b$. Then use the coordinates of one of the points in this equation and solve for b.

<u>Notes</u>

Guided Example   Practice

Find the equation of the line that contains the points (5, 11) and (3, 4).	1. Find the equation of the line that contains the points (2, 6) and (5, 18).
Solution The slope between the points is $$m = \frac{11-4}{5-3} = \frac{7}{2}$$ Putting this into the slope-intercept form of a line gives $$y = \tfrac{7}{2}x + b$$ To find b, use one of the points such as (3, 4) in place of x and y: $$4 = \tfrac{7}{2}(3) + b$$ $$4 = \tfrac{21}{2} + b$$ $$-\tfrac{13}{2} = b$$ This makes the equation through the points $$y = \tfrac{7}{2}x - \tfrac{13}{2}$$	

Guided Example   Practice

The Consumer Price Index (CPI) is a measure of the change in the cost of goods over time. If 1982 is used as the base year of comparison in some country (CPI = 100 in 1982), then the CPI of 205.6 in 2006 would indicate that an item that cost $1.00 in 1982 would cost $2.06 in 2006 in this country. It is known that the CPI in this country has been increasing at an approximately linear rate for the past 30 years.	2. Use 2000 as the base year, suppose that the CPI is 302 in 2010. a. Use this information to determine a linear function for this data, letting x be the years since 2000.

a. Use this information to determine a linear function for this data, letting x be the years since 1982.

   Solution To find the line for this problem, we need to identify two points on the line from the information in the problem. Since x represents the number of years since 1982, think of the two data points as (0, 100) and (24, 205.6) where the y values is the CPI. The slope between these points is

   $$m = \frac{205.6 - 100}{24 - 0} = 4.4$$

   Since (0, 100) is the vertical intercept, $b = 100$. Putting this information together gives
   $$y = 4.4x + 100$$

b. Based on your function, what was the CPI in 2000? Compare this estimate to the actual CPI of 181.4 for this country.

   Solution The year 2000 corresponds to $x = 18$. Substitute this value into the equation to get
   $$y = 4.4(18) + 100 = 179.2$$

   The difference between the estimated and actual CPI is 179.2 − 181.4 = -2.2. This is

   $$\frac{-2.2}{181.4} \approx -1.2\%$$

   of the actual CPI.

c. How is the annual CPI changing?

   Solution Since the slope of the line is 4.4, the CPI is increasing by 4.4 CPI units per year.

b. Based on your function, what was the CPI in 2017? Compare this estimate to the actual CPI of 440 for this country.

c. How is the annual CPI changing?

| | |
| | |

Question 4 -   How do you find cost and revenue functions?

Key Terms

None

Summary

Many of the functions in business and economics may be modeled with a linear function. For instance, suppose the price per unit for a product is fixed. The revenue from selling x units of the product is

$$R(x) = mx$$

where m is the price per unit. In this case, the revenue function is a linear function with slope m and vertical intercept 0. Since the price per unit is the increased revenue expected from selling each additional unit, it could also be called the marginal revenue.

Cost functions may also be linear functions. If the cost per unit is a constant m (the variable cost) and the fixed cost is b, the cost function is

$$C(x) = mx + b$$

Profit $P(x)$ is related to revenue and cost by

$$P(x) = R(x) - C(x)$$

If the revenue and cost function are both linear, the resulting profit function will also be linear.

Notes

Guided Example	Practice
Suppose the price of a USB flash drive sold is fixed at $21.	1. Suppose the price of a USB flash drive sold is fixed at $15.
a. Find the revenue $R(x)$ where x is the quantity of USB flash drives sold.	a. Find the revenue $R(x)$ where x is the quantity of USB flash drives sold.
Solution Revenue is related to price and quantity by	
$$\text{Revenue} = \text{Price} \cdot \text{Quantity}$$	
In this problem, revenue is represented by $R(x)$, Since the quantity is x and the price is $21, the revenue function is	
$$R(x) = 21x$$	
b. If the quantity sold is increased from 1500 to 1600, how much additional revenue is received?	b. If the quantity sold is increased from 1000 to 1200, how much additional revenue is received?
Solution To find how much the revenue has increased, we need to find the revenue at $x = 1500$ and $x = 1600$. By subtracting the revenue at these levels, we determine how much additional revenue is received:	
$$R(1600) - R(1500) = 21(1600) - 21(1500)$$ $$= 2100$$	
The additional revenue is $2100.	

Guided Example

Practice

| Write a linear cost function for a ski resort that charges a snow pass for $22 plus $52 per day.

Solution A linear cost function would have the form $C(t) = mt + b$. The independent variable is t and represents the number of days you get the snow pass for. This choice is due the rate, 52 dollars per day, has a denominator of day in the unit. The fixed cost of $22 is charged regardless of how many days the now pass if for and is the vertical intercept. Putting these numbers together in the cost function leads to

$$C(t) = 52t + 22$$ | 2. Write a linear cost function for a manufacturer that charges $2200 plus $480 per unit. |

Guided Example

Practice

| A manufacturer spends $52000 to produce 130 parts, achieving a marginal cost of $75 per part. Find the linear cost function.

Solution In this problem we'll use a linear function of the form $C(x) = mx + b$ where x is the number of part produced. The marginal cost gives us the value of m,

$$C(x) = 75x + b$$

To find b, we need to realize that 130 parts cost $52000 or $C(130) = 52000$. By substituting $x = 130$ into the function and setting the resulting equation equal to 52000, we get | 3. A manufacturer spends $120000 to produce 150 parts, achieving a marginal cost of $520 per part. Find the linear cost function. |

$$75(130) + b = 52000$$
$$9750 + b = 52000$$
$$b = 42250$$

Putting this vertical intercept and the slope into a linear function form yields

$$C(x) = 75x + 42250$$

Question 5 - What are demand and supply?

<u>Key Terms</u>

Demand   Supply

Shortage   Surplus

<u>Summary</u>

Supply and demand functions are functions that relate the unit price of a product to a quantity. In a supply function $p = S(q)$, the unit price p is related to the quantity q of a product that a supplier would want to supply at that price. In many situations, the formula for this function will be linear with a positive slope. This is due to the fact that a supplier will want to supply more product as the price increases.

Demand functions $p = D(q)$ relate the unit price p with the quantity q demanded by consumers. In the simplest situations, the demand function is linear with a negative slope. This is due to the fact that most consumers will demand less of a product as the price grows.

The equilibrium point for a market is where the supply function is equal to the demand function, $S(q) = D(q)$. At this price per unit, the quantity supplied is equal to the quantity demanded. At unit prices above or below the equilibrium price the quantities supplied and demanded will be different. If the quantity supplied is greater than the quantity demanded, there is a surplus in the market. If the quantity demanded is greater than the quantity supplied, there is a shortage in the market.

<u>Notes</u>

Guided Example

Practice

An electronics manufacturer makes a streaming stick that allows consumers to view content on the Internet. The unit price of the stick is $$D(q) = -0.5q + 75$$ where q is in thousands of sticks. The price per unit at which the manufacturer will supply the stick is $$S(q) = 0.25q$$ a. Find the equilibrium point. **Solution** The equilibrium point is where the demand is equal to the supply, $D(q) = S(q)$. Set the two functions equal and solve for q to get the equilibrium quantity: $$-0.5q + 75 = 0.25q$$ $$75 = 0.75q$$ $$100 = q$$ The corresponding equilibrium price may be calculated from either function, $$D(100) = -0.5(100) + 75 = 25$$ $$S(100) = 0.25(100) = 25$$ The equilibrium quantity is 100000 sticks at a price per unit of $25. b. Does a price per unit of $50 correspond to a surplus or shortage?	1. A small chocolate manufacturer makes and sells boxes of chocolate. The unit price of a box is $$D(q) = -0.05q + 6$$ where q is in hundreds of boxes. The price per unit at which the manufacturer will supply the boxes is $$S(q) = 0.1q$$ a. Find the equilibrium point. b. Does a price per unit of $3 correspond to a surplus or shortage?

Solution Calculate the corresponding quantities for demand and supply: $\quad -0.5q + 75 = 50 \qquad 0.25q = 50$ $\quad\quad -0.5q = -25 \qquad\quad q = 200$ $\quad\quad\quad q = 50$ At this price, consumers demand 50000 units and suppliers wish to supply 200000. This indicates that there is a surplus of 150000 units.	

Section 1.3 Approximate Linear Models

Question 1 - Which linear model is best?

Question 2 - How do you use linear regression functions?

Question 3 - How good is the linear model?

Question 1 - Which linear model is best?

Key Terms

Line of best fit   Linear regression

Summary

To find the line of best fit, we'll do linear regression using technology. Desmos is a free online app that is easy to use. View the Youtube video (https://youtu.be/QFjKObVJjM4) to help you perform linear regression. You can also use a graphing calculator or Google Sheets to carry out linear regression. Consult your classes technology resources for help on using those tools.

Linear regression finds a line of the form $y = mx + b$ that passes "closest" to the data points. By "closest", we mean that the square of the vertical distance between the points and line are as small as possible. When using a linear regression tool, you will need to identify the value of m and b in order to give the equation of the line of best fit.

Some forms of technology define the line to have a form $y = ax + b$. In this case, linear regression will return the values of a and b on the line of best fit.

Notes

Guided Example

The table shown below gives the national average pupil-teacher ratio in public schools over selected years.

Year	Ratio
1992	19.4
1996	19.7
2000	18.9
2004	18.2
2008	16.8

Find the equation for the least squares line. Let x correspond to the number of years since 1992 and let y correspond to the average number of pupils per 1 teacher.

Solution Enter the data into Desmos (desmos.com) as demonstrated in the video linked in the summary above.

You should end up with a screen like the one below.
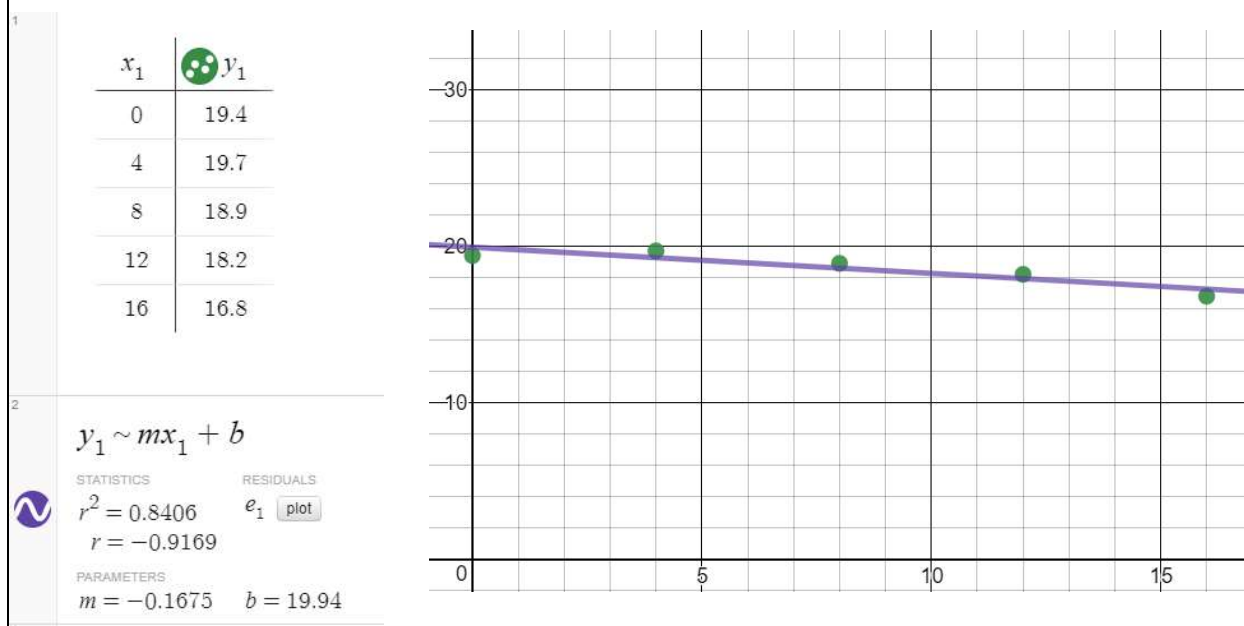
Take special note of the data values in the first column. The x values (in this case called x_1) are described as the number of years since 1992. This means that 1992, 1996, 2000, … corresponds to 0, 4, 8, …

Examining the screen above, we see that $m = -0.1675$ and $b = 19.94$. This means the equation of the least squares line is

$$y = -0.1675x + 19.94$$

Practice

1. The table shown below gives the average cost for a year of public four-year college in the US for several years

Year	Cost ($)
2010	7415
2011	7947
2012	8183
2013	8266
2014	9139

Find the equation for the least squares line. Let x correspond to the number of years since 2000 and let y correspond to the average cost in dollars.

Question 2 - How do you use linear regression functions?

<u>Key Terms</u>

None

<u>Summary</u>

Once a line of best fit is found for a set of data, you may be asked questions about the line of best fit. For instance, you may be given a value for the independent variable and asked to calculate the dependent variable. You may also be given a value for the dependent variable and asked to solve for the independent variable. In either case, pay careful attention to the units on your variable to make sure you put in the appropriate values for the variables.

In some problems, you may be asked to interpret the meaning of the slope. As in earlier questions, use the units on the independent variable and dependent variable to determine the units on the slope,

$$\text{units on slope} = \frac{\text{units on dependent variable}}{\text{units on independent variable}}$$

A good interpretation of the slope will include the number, the units on the slope, and a description of how the variables are changing with respect to each other. For instance, if the slope on line of best fit modeling cost is 2 dollars per case, we might say, "each additional case increases cost by 2 dollars".

<u>Notes</u>

Guided Example

Practice

In the previous guided example, you were given the data below.

Year	Ratio
1992	19.4
1996	19.7
2000	18.9
2004	18.2
2008	16.8

The line of best fit was found to be

$$y = -0.1675x + 19.94$$

where x is the number of years since 1992 and y is the ratio.

a. Use the line of best fit to predict the pupil-teacher ratio in 2020.

   Solution The year 2020 corresponds to $x = 28$. To find the ratio in that year, substitute $x = 28$ into the line of best fit:

   $$y = -0.1675(28) + 19.94 = 15.25$$

b. According to the model (line of best fit), when will the pupil-teacher ratio reach 12?

   Solution The pupil-teacher ratio of 12 corresponds to $y = 12$. To find the corresponding year, set $y = 12$ in the line of best fit and solve for x:

1. In the previous practice problem, you were given the data below.

Year	Cost ($)
2010	7415
2011	7947
2012	8183
2013	8266
2014	9139

The line of best fit was found to be

$$y = 376.7x + 3669.6$$

where x is the number of years since 2000 and y is the average cost of four-year college.

a. Use the line of best fit to predict the average cost in 2018.

b. According to the model (line of best fit), when will the average cost reach $15,000?

|	
$12 = -0.1675x + 19.94$ $-7.94 = -0.1675x$ $47.4 \approx x$ The ratio will reach 12 in the year 2039. c. What does the slope of the line of best fit tell you about the pupil-teacher ratio? **Solution** Each year the pupil-teacher ratio decreases by .1675.	c. What does the slope of the line of best fit tell you about average college costs??

Question 3 - How good is the linear model?

<u>Key Terms</u>

Correlation coefficient

<u>Summary</u>

The correlation coefficient is an indicator of how well two variables are correlated. The value of the correlation coefficient, r, ranges from -1 to 1. Negative correlation coefficients indicate a scatter plot where the trend is decreasing. A positive correlation coefficient indicates a scatter plot where the trend is increasing.
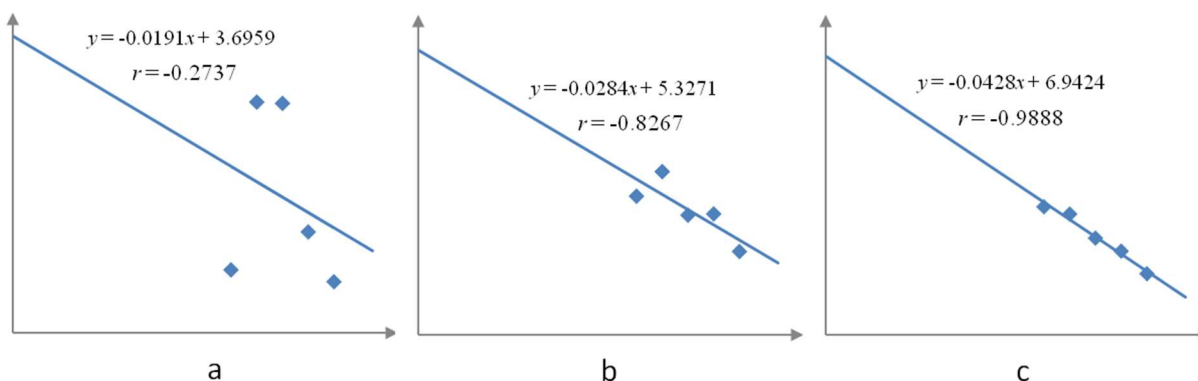
If the absolute value of r is close to 1, the data values lie close to the line of best fit (as in c). In this case we would say the variables are strongly correlated. If the absolute value of r is close to zero, the data values will be spread out from the line of best fit (as in a). In this case, we would say the variables are weakly correlated.

<u>Notes</u>

Guided Example	Practice

Guided Example

In the previous guided examples, you were given the data below.

Year	Ratio
1992	19.4
1996	19.7
2000	18.9
2004	18.2
2008	16.8

The line of best fit was found to be

$$y = -0.1675x + 19.94$$

with correlation coefficient $r = -0.9166$.

What does the correlation coefficient tell you about the line of best fit?

Solution The correlation coefficient is negative so the line of best fit has a negative slope. The closer $|r|$ is to 1, the stronger the linear correlation. The correlation coefficient is -0.9166 and indicates a strong linear correlation.

Practice

1. In the previous practice problem, you were given the data below.

Year	Cost ($)
2010	7415
2011	7947
2012	8183
2013	8266
2014	9139

The line of best fit was found to be

$$y = 376.7x + 3669.6$$

with correlation coefficient $r = 0.9519$

What does the correlation coefficient tell you about the line of best fit?

Chapter 1 Solutions

Section 1.1

Question 1   1) a. $P=-\frac{1}{2}Q+3$, b. $Q = -2P + 6$

Question 2   1) a. linear function, b. not linear function, c. not linear function

2) $Q=-\frac{2}{3}P+120$, b. $m=-\frac{2}{3}\approx-0.66$.

Question 3   1) a. 24, b. -4, c. $3z-6$, d. 12

2) a. 345, b. 50

Section 1.2
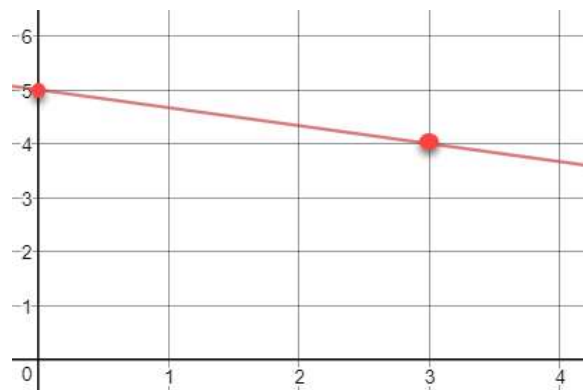
Question 1   1) a. $m=\frac{1}{3}$, $b = -4$ b. $m=-\frac{1}{2}$, $b = 3$ c. $m=-\frac{1}{4}$, $b = 2$

2) $y = 2x - 2$

3) $y = 3$

4) a. $-\frac{3}{2}$, b. Lowering the price by approximately 1.5 dollars increases demand by 1 unit, c. $b=180$.

Question 2   1)

Question 3   1) $y = 4x - 2$

2) a. $y = 20.2x + 100$, b. 443.4, 0.8% higher than actual CPI of 440, c. Increasing at a rate of 20.2 per year.

Question 4   1) a. $R(x) = 15x$, b. $R(1200) - R(1000) = 3000$

2) $C(x) = 480x + 2200$

3) $C(x) = 520x + 42000$

Question 5   1) a. 4000 boxes at a price per unit of $4, b. shortage

Section 1.3

Question 1   1) $y = 376.7x + 3669.6$

Question 2   1) a. $10450.20, b. 2030, c. Each year the average college cost increases by $376.70.

Question 3   1) The correlation coefficient is 0.9519 and indicates a strong linear correlation.