Question 1: How do you find the probability of an event?

An event is any collection of outcomes in the sample space. The probability of an event is the sum of the probabilities of the outcomes corresponding to the event.

Probability of an Event

If E is composed of a collection of the outcomes in the sample space S,

$$E = \{e_1, e_2, \ldots, e_n\}$$

then the probability of the event E is

$$P(E) = P(e_1) + P(e_2) + \cdots + P(e_n)$$

We can use the probabilities of the outcomes in the sample space to find the probability of any event.

Example 1 Probability of an Event

A market research firm conducted a survey of smartphone users to determine their data consumption. The table below indicates the amount of data used in one month.

Amount of Data Used	Number of Users
Less than 200 MB	1291
200 MB up to, but not including 500 MB	758
500 MB up to, but not including 1 GB	423
1 GB up to, but not including 2 GB	324
2 GB or more	947

a. If each row corresponds to an outcome from an experiment, find the probability of each outcome.

Solution We will use relative frequencies to estimate the probability of each outcome. Define the outcomes as

e_1 : Less than 200 MB of data used

e_2 : 200 MB up to, but not including 500 MB of data used

e_3 : 500 MB up to, but not including 1 GB of data used

e_4 : 1 GB up to, but not including 2 GB of data used

e_5 : 2 GB or more of data used

The total number of users is $1291+758+423+324+947=3743$. The probability of each outcome is

$$P(e_1)\approx\frac{1291}{3743}\approx 0.345$$
$$P(e_2)\approx\frac{758}{3743}\approx 0.203$$
$$P(e_3)\approx\frac{423}{3743}\approx 0.113$$
$$P(e_4)\approx\frac{324}{3743}\approx 0.087$$
$$P(e_5)\approx\frac{947}{3743}\approx 0.253$$

The sum of decimals are slightly higher than 1. This is due to rounding and is to be expected. However, the sum of the fractions are 1 as they should be.

b. Find the probability that a user will use 1 GB or more of data.

Solution Let A be the event "1 GB or more of data used". Since A is composed of the outcomes e_4 and e_5,

$$\begin{aligned} P(A) &= P(e_4) + P(e_5) \\ &\approx 0.087 + 0.253 \\ &\approx 0.340 \end{aligned}$$

c. Find the probability that a user will use 200 MB up to, but not including 2 GB of data.

Solution Let B be the event "200 MB up to, but not including 2 GB of data used". This event is made up of outcomes e_2, e_3 and e_4. The probability of the event B is

$$\begin{aligned} P(B) &= P(e_2) + P(e_3) + P(e_4) \\ &\approx 0.203 + 0.113 + 0.087 \\ &\approx 0.403 \end{aligned}$$

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In the previous example, the outcomes from the experiment were estimated using relative frequencies. The probability of events composed of equally likely outcomes may be calculated by counting the number of outcomes in the event. For instance, if an event contains M outcomes(each with probability $\frac{1}{N}$), the probability of the event must be

$$P(E) = \underbrace{\frac{1}{N} + \frac{1}{N} + \cdots + \frac{1}{N}}_{M \text{ terms}} = \frac{M}{N}$$

Probability of an Event With Equally Likely Outcomes

Suppose the sample space of an experiment contains N equally outcomes. If an event E contains M of those outcomes, the probability of the event is

$$P(E) = \frac{M}{N}$$

This expression must be used cautiously since it requires that each outcome in the sample space be equally likely. This expression is often written using the letter n to indicate the number of outcomes in a collection. In this case,

$$P(E) = \frac{n(E)}{n(S)}$$

where the notations $n(E)$ and $n(S)$ are the number of outcomes in the event E and sample space S.

Example 2 Probability of an Event

A company supplying bicycle parts ships parts from warehouses located in Newark, New Jersey, Jacksonville, Florida, Industry, California, Portland, Oregon, and Dallas, Texas. Based on past history, the shipping manager has determined that the likelihood of an order being fulfilled by a particular warehouse is the same as being supplied by any other warehouse.

a. Find the probability that an order is fulfilled by the warehouse in Industry, California.

Solution Let's consider this to be an experiment where the outcomes are the location where an order is fulfilled. This means the sample space for the experiment is

$$S = \{\text{Newark, Jacksonville, Industry, Portland, Dallas}\}$$

Based on past history, each of these outcomes is just as likely as any other. The event that the order is fulfilled by the Industry warehouse is one outcome of five in the sample space, so

$$P(\text{Industry}) = \frac{1}{5} = 0.2$$

There is a 20% chance that an order will be fulfilled by the Industry warehouse.

b. Find the probability that an order will be fulfilled by a warehouse east of the Mississippi River.

Solution This event corresponds to the outcomes Newark and Jacksonville. Since this event contains 2 outcomes from the sample space,

$$P(\text{Newark, Jacksonville}) = \frac{2}{5} = 0.4$$

There is a 40% chance that the order will be fulfilled by a warehouse east of the Mississippi River.

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