Question 3: How do you find the probability of a compound event?

Events may be combined together in various ways. These combinations are called compound events. If we know the probability of the events that make up the compound event, we are often able to compute the probability of the compound event.

Outcomes that are in the event A as well as the event B are said to be in the compound event, A and B. The word "and" is used to indicate that the outcomes in this event are in both events simultaneously. Mathematicians describe outcomes in A and B with the intersection symbol $\cap$. An outcomes in A and B are the same outcomes in the intersection of A with B, $A \cap B$. The probability of A and B occurring is often referred to as the joint probability of A and B.

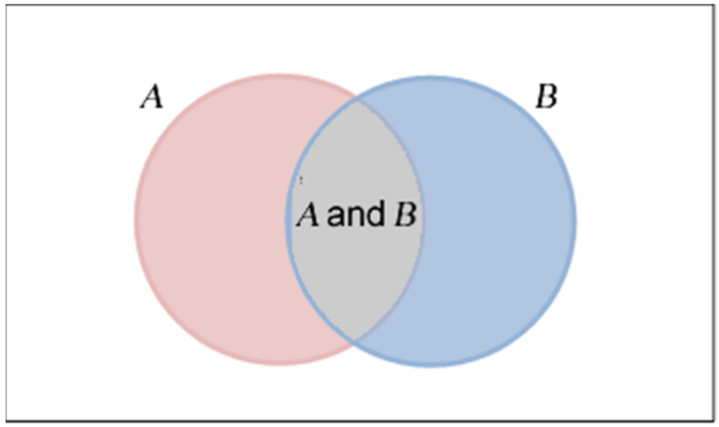
Another type of compound event is denoted using the word "or". An outcome is in the event A or B if it is in A, B, or both events simultaneously. In the language of sets, the compound event A or B is the same as the union of the set A with the set B. The symbol $\cup$ represents the union of two sets. Using this symbol, we write the union of A with B as $A \cup B$. In this text we will use the word "or" instead of the union symbol to represent outcomes in A, in B, or in both events.

Example 4 Compound Events

Suppose A is the event "consumer plans to purchase a laptop computer in the next six months" and B is the event "consumer plans to purchase a tablet computer in the next six months". Describe the outcomes in each of the events below.

a. A and B

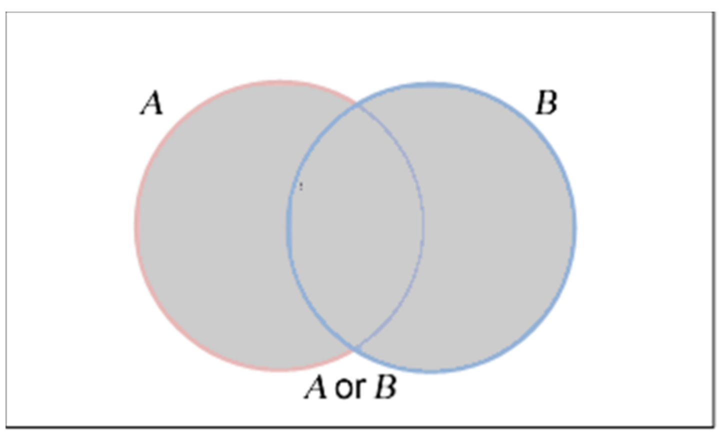
Solution The event A and B corresponds to outcomes that are in both A and B. In other words, A and B is "consumer plans to purchase a laptop and a tablet computer in the next six months". To get a visual sense of these outcomes, imagine the consumers in each event as being represented by circles.



Outcomes in *A* and *B* are in the gray region where both circles overlap.

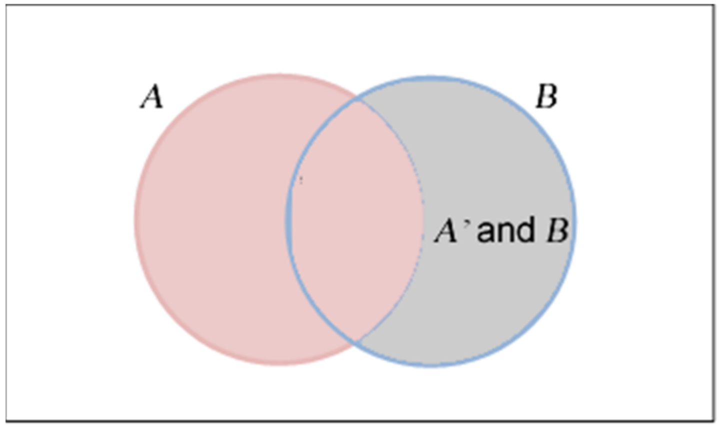
b. *A* or *B*

Solution The event A or B corresponds to outcomes that are in A, in B, or in both events. The phrase “consumers who plan to purchase a laptop computer or a tablet computer in the next six months” describes these outcomes. The gray region below gives a visual representation of these consumers.



c. *A*’ and *B*

Solution The outcomes in *A*’ and *B* must be in both events. This means that the consumers do not plan to purchase a laptop computer, but do plan to purchase a tablet computer. Visually, this event is the gray region outside of *A* and inside of *B*.



The probability of the events A and B and the probability of the events A or B are related to each other. We can see how this relationship works by examining a survey of 200 consumers.

Event	**Description**	**Number of consumers**
A	Consumers who plan purchase a laptop computer in the next six months	23
B	Consumers who plan to purchase a tablet computer in the next six months	17
A and *B*	Consumers who plan to purchase a laptop computer and a tablet computer in the next six months.	5

To calculate the probability of the event A or B, we need to use the survey to count the number of consumers in the event A or B. Initially you might try to simply add the consumers in A and the consumers in B. However, since there are 5 consumers in both sets, A and B, those consumers would be counted twice. To fix this problem, we add the number of consumers in A to the number of consumers in B, and subtract the consumers in A and B:

$$\underbrace{23}_{n(A)} + \underbrace{17}_{n(B)} - \underbrace{5}_{n(A \text{ and } B)} = \underbrace{35}_{n(A \text{ or } B)}$$

Divide each term by the number of consumers in the sample space, 200, to get the probability of each event.

$$\underbrace{\frac{23}{200}}_{P(A)} + \underbrace{\frac{17}{200}}_{P(B)} - \underbrace{\frac{5}{200}}_{P(A \text{ and } B)} = \underbrace{\frac{35}{200}}_{P(A \text{ or } B)}$$

The likelihood of a consumer planning to purchase a laptop computer or a tablet computer in the next six months is $\frac{35}{200}$ or 17.5%. This relationship holds even when the outcomes in the sample space are equally likely or not equally likely.

The Probability of A or B

The likelihood of an event A occurring or an event B occurring is

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If we know the individual probabilities $P(A)$ and $P(B)$, as well as one of the compound events, this relationship may be used to find the other compound event.

Example 5 Probability of a Compound Event

In Example 2 we examined probabilities associated with a company shipping bicycle parts from warehouses in Newark, New Jersey, Jacksonville, Florida, Industry, California, Portland, Oregon, and Dallas, Texas. Suppose we define

A: “part ships from east of the Mississippi”

B: “part ships from a coastal state”

Find the probability the part ships from east of the Mississippi or from a coastal state. Assume each outcome is equally likely.

Solution Recall that each outcome in the sample space is equally likely. The likelihood of each event is

$$P(A) = \frac{2}{5} = 0.4$$
$$P(B) = \frac{4}{5} = 0.8$$

Additionally, there are two outcomes in the event "part ships from east of the Mississippi" and "part ships from a coastal state". This means

$$P(A \text{ and } B) = \frac{2}{5} = 0.4$$

Using these values, the probability of the event "part ships from east of the Mississippi" or "part ships from a coastal state" is

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 0.4 + 0.8 - 0.4 \\ &= 0.8 \end{aligned}$$

In Example 4, we could also simply count the number of outcomes in the event "part ships from east of the Mississippi" and "part ships from a coastal state". Since there are 4 outcomes, Newark, New Jersey, Jacksonville, Florida, Industry, California, and Portland, Oregon, in the compound event, we could also calculate the probability as $\frac{4}{5}$.

Counting outcomes is often impractical for experiments with larger number of outcomes.