Question 1: What is conditional probability?

A typical consumer survey might result in data like that shown below.

Type of Phone Owned	Male Consumers	Female Consumers	Total for each row
Basic Phone	651	660	1311
Smart Phone	1411	1021	2432
Total for each column	2062	1681	3743

Based on what we have done in previous sections, we could calculate the likelihood that a consumer in the survey uses a smartphone. From the table we know there are 2432 smartphone owners in the 3743 consumers who were surveyed. Using relative frequencies, the probability of a consumer owning a smartphone is

$$P(\text{consumer owning a smartphone}) = \frac{2432}{3743} \approx 0.650$$

A marketer might be interested in knowing whether the fact that a consumer is male changes the likelihood that they own a smartphone. In other words, given that the consumer is male, what is the likelihood that the consumer owns a smartphone. This is an example of conditional probability. In conditional probability, one event is assumed to have occurred and we are interested in knowing the likelihood of another event occurring.

If we define

M: consumer is male

S: consumers owns a smartphone

as the two events. We would like to determine the probability that S occurs given that M has occurred. This probability is represented using a vertical bar and is written $P(S \mid M)$ .This is read “the probability of S occurring given that M has occurred”

We can calculate this probability using the values in the table.

Type of Phone Owned	Male Consumers	Female Consumers	Total for each row
Basic Phone	651	660	1311
Smart Phone	1411	1021	2432
Total for each column	2062	1681	3743

Since we know the consumer is a male, we now constrain ourselves to the data in the column shaded above. We are no longer considering all 3743 consumers in the survey. Now we are only examining the 2062 males who took the survey. Of those males, 1411 own a smartphone. This gives us the conditional probability

$$P(S \mid M) = \frac{1411}{2062} \approx 0.684$$

In this relative frequency, the numerator is the number of consumers in the compound event "consumer is male and consumer owns a smartphone". The denominator is the number of consumers in the event "consumer is male". For this pair of events, assuming that the consumer is male makes the likelihood that they own a smartphone slightly higher than if we do not make this assumption.

Example 1 Conditional Probability

Let F and B represent the events,

F: consumer is female

B: consumer owns a basic phone

Use the data from the consumer survey below to find the conditional probabilities in each part.

Type of Phone Owned	Male Consumers	Female Consumers	Total for each row
Basic Phone	651	660	1311
Smart Phone	1411	1021	2432
Total for each column	2062	1681	3743

a. The probability that a consumer owns a basic phone given that the consumer is female.

Solution There are 1681 female consumers in the survey. Of this amount, 660 own a basic phone. The likelihood a consumer owns a basic phone given that the consumer is female is

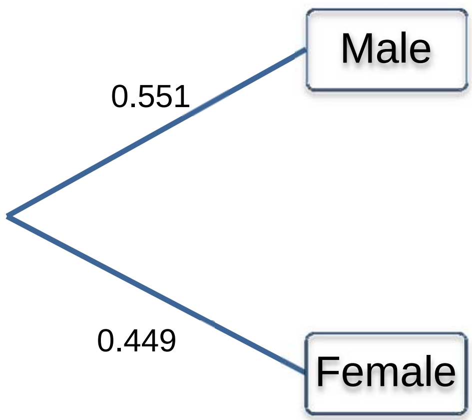
$$P(B \mid F) = \frac{660}{1681} \approx 0.393$$

b. The probability that a consumer is female given that the consumer owns a basic phone.

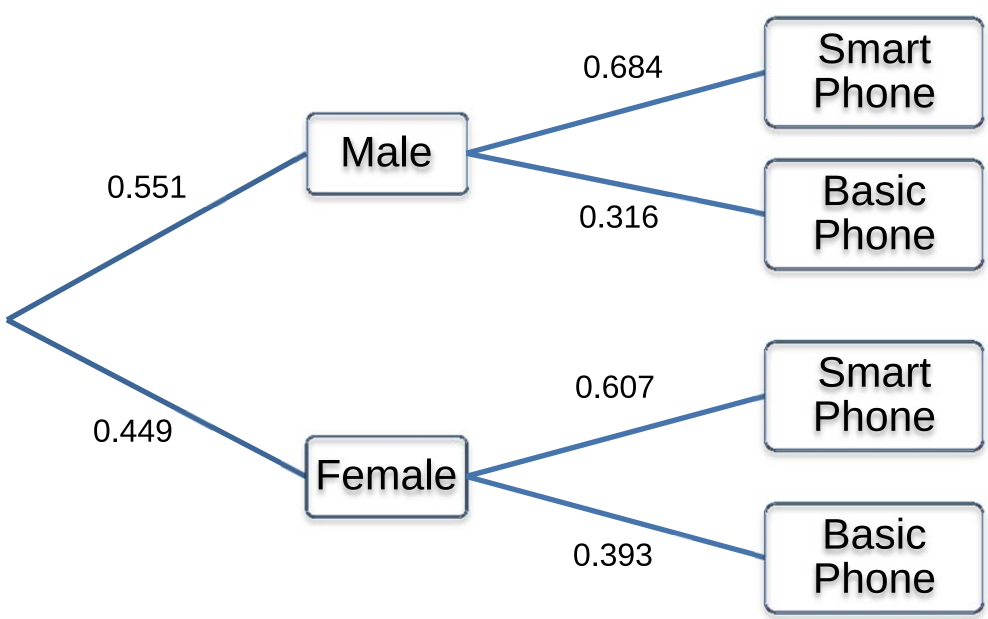
Solution Start with the 1311 consumers who own a basic phone. Of these consumers, 660 are female. The corresponding relative frequency gives the conditional probability,

$$P(F \mid B) = \frac{660}{1311} \approx 0.503$$

Conditional and marginal probabilities are often represented pictorially using a tree diagram. In a tree diagram, branches grow from branches and help to identify various conditional probabilities. For instance, we might start a tree diagram with two branches indicating whether the consumer is male or female. The probabilities for “consumer is male” and “the consumer is female” are written along the corresponding branch.



From each of these branches, we branch to smart phone or basic phone. Each of these branches represents a conditional probability. For instance, the branch originating at male and ending at smart phone represents the probability that the consumer owns a smart phone given the consumer is a male. The point at which the branch originates establishes the event that has occurred. The event at which the branch ends establishes what probability we are interested in.



For each set of branches, the sum of the probabilities is equal to 1. For instance,

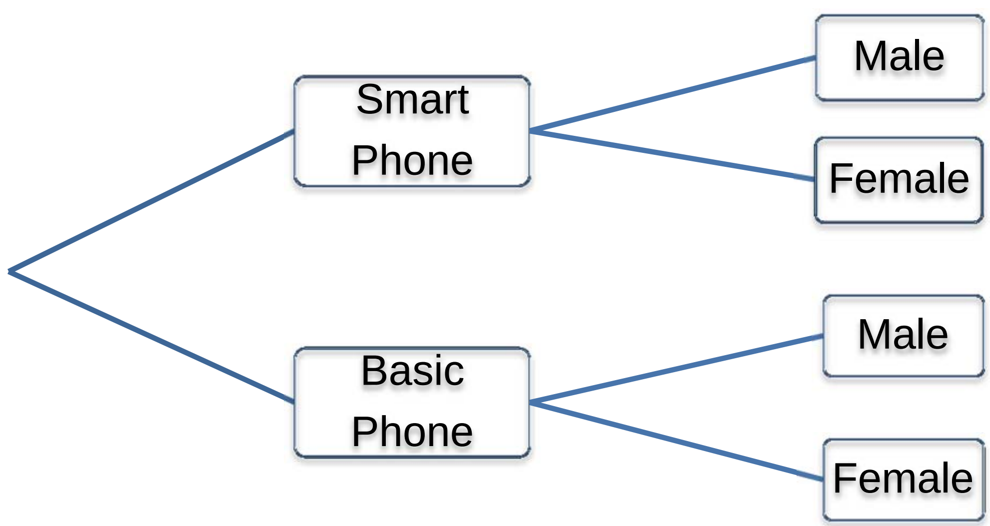
$$P(S \mid F) + P(B \mid F) \approx 0.607 + 0.393 = 1$$

This characteristic allows you to check the probabilities quickly to insure a simple arithmetic error has not been made.

Example 2 Tree Diagrams

Type of Phone Owned	Male Consumers	Female Consumers	Total for each row
Basic Phone	651	660	1311
Smart Phone	1411	1021	2432
Total for each column	2062	1681	3743

Use the results of the consumer survey above to label each of the braches on the tree diagram below.



Solution The first branches are to the events

S: consumer owns a smart phone

B: consumer owns a basic phone

The probability of these events are using the numbers of consumers in each event,

$$P(S) = \frac{2432}{3743} \approx 0.650$$
$$P(B) = \frac{1311}{3743} \approx 0.350$$

As we hoped, $P(S) + P(B) = \frac{2432}{3743} + \frac{1311}{3743} = 1$.

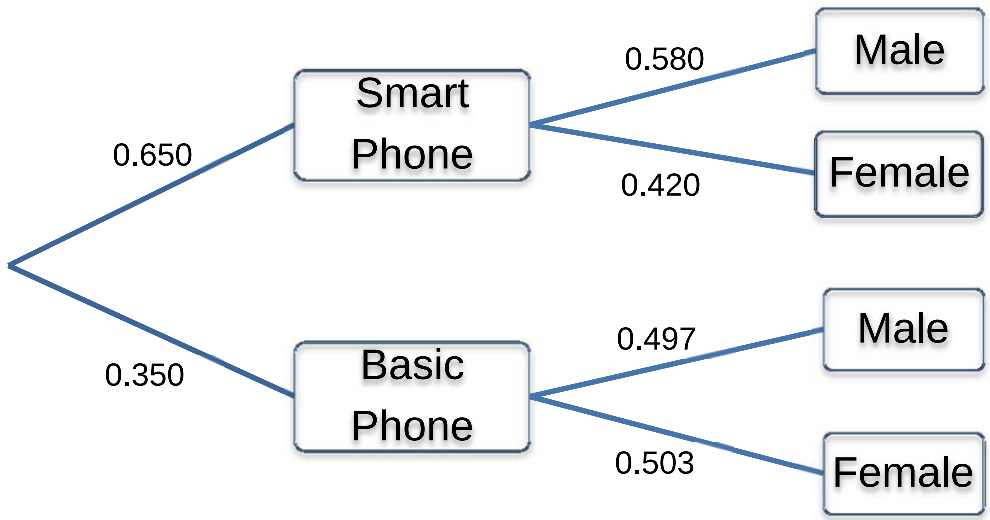
Next, calculate the conditional probabilities originating from the event "consumer owns a smart phone". From the survey, we know 2432 consumers own a smart phone. Since 1411 of those owners are male and 1021 are female,

$$P(M \mid S) = \frac{1411}{2432} \approx 0.580$$
$$P(F \mid S) = \frac{1021}{2432} \approx 0.420$$

The conditional probabilities originating from the event "consumer owns a basic phone" are calculated in a similar manner. Of the 1311 consumers who own a basic phone, 651 are male and 660 are female. This gives the probabilities

$$P(M \mid B) = \frac{651}{1311} \approx 0.497$$
$$P(F \mid B) = \frac{660}{1311} \approx 0.503$$

Label each of these probabilities on the tree diagram.


0.580
Male
Smart
Phone
0.650
0.420
Female
0.497
Male
Basic
Phone
0.350
0.503
Female