Question 2: How is conditional probability computed?

Let’s look at the expressions we have used to compute conditional probability. To compute the likelihood that a consumer is female given that the consumer owns a basic phone, we computed the relative frequency

$$P(F \mid B) = \frac{660}{1311} \approx 0.503$$

The numbers in this relative frequency came from a the consumer survey summarized in the table below.

Type of Phone Owned	Male Consumers	Female Consumers	Total for each row
Basic Phone	651	660	1311
Smart Phone	1411	1021	2432
Total for each column	2062	1681	3743

Since we know that the consumer owns a basic phone, the denominator is the total number of consumers that own a basic phone, 1311. The numerator is the number of consumers who own a basic phone and who are female, 660. Note the use of the word “and”. This number is the number of consumers in the joint event F and B. Using the letter n to denote the number of items in the collection, we write

$$P(F \mid B) = \frac{\overbrace{660}^{n(F \text{ and } B)}}{\underbrace{1311}_{n(B)}}$$

This fraction is equivalent to

$$P(F \mid B) = \frac{\overbrace{\frac{660}{3743}}^{\frac{n(F \text{ and } B)}{n(S)}}}{\underbrace{\frac{1311}{3743}}_{\frac{n(B)}{n(S)}}}$$

In this expression, the numerator is divided by the total number of consumers in the survey, 3743. In terms of probabilities, the fraction on top is the probability $P(F \text{ and } B)$. The bottom is also a probability, $P(B)$. If we put these probabilities into the conditional probability, we get

$$P(F \mid B) = \frac{P(F \text{ and } B)}{P(B)}$$

This expression allows us to compute the conditional probability from the joint and marginal probabilities.

Conditional Probability

If A and B are events, the likelihood of A occurring given that B has occurred is

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

provided that $P(B) \neq 0$.

In this expression, the probability in the denominator is always the probability of the given event. Since its probability is never zero, we know the event will actually occur.

Example 3 Conditional Probability

A community college is interested in hiring qualified instructors to teach online courses. The community college estimates that the likelihood that a candidate will have the proper educational background is is 0.8. The probability that a candidate has online teaching experience and proper educational background is 0.1. If a candidate is randomly selected and is found to have the proper educational background, what is the likelihood that they have online teaching experience?

Solution Define two events for this application,

A: candidate has online teaching experience

B: candidate has the proper educational background

The information in the application gives us two probabilities,

$$P(B) = 0.8$$
$$P(A \text{ and } B) = 0.1$$

The likelihood of a candidate having online teaching experience given they have the proper educational experience is

$$\begin{aligned} P(A \mid B) &= \frac{P(A \text{ and } B)}{P(B)} \\ &= \frac{0.1}{0.8} \\ &= 0.125 \end{aligned}$$

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