Question 3: What are independent events?

When computing conditional probabilities, you might be curious to know whether the fact that an event has occurred has any effect on the probability of another event. For instance, earlier in this section we computed the probability of a consumer owning a smartphone as $P(S) \approx 0.650$. We also computed the likelihood of a consumer owning a smartphone given the fact that the consumer is male, $P(S \mid M) \approx 0.684$. The fact that the consumer is male changes the likelihood of the consumer owning a smartphone. These events are an example of dependent events since one event occurring changes the likelihood of another event occurring. When two events are independent, one event occurring has no effect on the likelihood of the other event occurring.

Independent Events

If one vent occurring does not change the likelihood of another event occurring, the two events are independent. This means that for events A and B,

$$P(A \mid B) = P(A) \text{ and } P(B \mid A) = P(B)$$

Example 4 Are the Events Independent?

In Example 1, we calculated the conditional probabilities

$$P(B \mid F) = \frac{660}{1681} \approx 0.393$$
$$P(F \mid B) = \frac{660}{1311} \approx 0.503$$

where F and B represent the events,

F: consumer is female

B: consumer owns a basic phone

Are F and B independent events?

Solution We already have the conditional events. To determine if $P(B|F) = P(B)$ and $P(F|B) = P(F)$, we need to compute $P(B)$ and $P(F)$. The information for these events are given in the table below.

Type of Phone Owned	Male Consumers	Female Consumers	Total for each row
Basic Phone	651	660	1311
Smart Phone	1411	1021	2432
Total for each column	2062	1681	3743

The likelihood of “consumer is female” is calclated by dividing the number of female consumers by the number of consumers surveyed,

$$P(F) = \frac{1681}{3743} \approx 0.449$$

Since $P(F|B) = \frac{660}{1311} \approx 0.503$, the likelihood of F changes if B has occurred. Therefore, F and B are dependent.

Similarly, the likelihood that the “consumer owns a basic phone” is

$$P(B) = \frac{1311}{3743} \approx 0.350$$

This is different from the conditional probability $P(B|F) = \frac{660}{1681} \approx 0.393$.

To prove that two events like B and F are dependent, we only need to show that $P(B|F) \neq P(B)$ or $P(F|B) \neq P(F)$. However, to prove that the events are independent, we must prove that both $P(B|F) = P(B)$ and $P(F|B) = P(F)$.