For a certain sports utility vehicle,

$$M(x) = -0.015x^2 + 1.31x - 7.3, \quad 30 \le x \le 60,$$

represents the miles per gallon obtained at a speed of x miles per hour. Find the absolute maximum miles per gallon and the absolute minimum miles per gallon, and the speeds they occur at.

Since this function is continuous and defined on a closed interval (one including the endpoints), we only need evaluate the function at the endpoints and any critical points to find the very highest point and the very lowest point on the function.

Find the Critical Points

To find the critical points, we need to find where the derivative of $M(x)$ is equal to zero or undefined. The derivative of $M(x)$ is

$$M'(x) = -0.03x + 1.31$$

This derivative is linear so is defined everywhere. The only critical points come from setting $M'(x)$ equal to 0:

$$-0.03x + 1.31 = 0$$

$$-0.03x = -1.31$$

$$x = \frac{1.31}{0.03} \text{ or } \frac{131}{3}$$

The equivalent fraction is found by multiplying the top and bottom decimals by 100. Note that this value is about 47 and falls into the clsed interval

Evaluate the Function

We have three possibilities for the absolute maximum and absolute minimum on this function: the endpoints $x = 30, 60$ or the critical point at $x = \frac{131}{3}$. To see which is highest and which is lowest, put these values into the *original* function $M(x)$. We use the original function because it returns y-values with which we can determine the highest and lowest points.

$$M(30) = -0.015(30)^2 + 1.31(30) - 7.3 = 18.5 \text{ mpg}$$

$$M\left(\tfrac{131}{3}\right) = -0.015\left(\tfrac{131}{3}\right)^2 + 1.31\left(\tfrac{131}{3}\right) - 7.3 \approx 21.3 \text{ mpg} \quad \text{Absolute Maximum}$$

$$M(60) = -0.015(60)^2 + 1.31(60) - 7.3 = 17.3 \text{ mpg} \quad \text{Absolute Minimum}$$