

Suppose the total cost  $C(x)$  (in dollars) to manufacture a quantity  $x$  of weed killer (in hundreds of liters) is given by

$$C(x) = x^3 - 2x^2 + 8x + 50$$

Where is  $C(x)$  decreasing?

**Find the derivative of  $C(x)$ :** Take the derivative of each term with the power rule:

$$C'(x) = 3x^2 - 4x + 8$$

**Find the critical numbers:** To find the critical numbers, set the derivative equal to zero and solve for  $x$ .

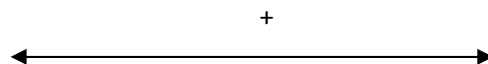
$$3x^2 - 4x + 8 = 0$$

To solve this quadratic equation, use the quadratic formula with  $a = 3$ ,  $b = -4$  and  $c = 8$ :

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(8)}}{2(3)} \\ &= \frac{4 \pm \sqrt{-80}}{6} \end{aligned}$$

This has no real solution so there are no critical numbers.

**Make a number line to track the derivative:** Normally we test around the critical numbers, but in this case there are none. By testing the derivative in any place, we get the sign of the derivative everywhere.



The easiest place to test is at 0, so evaluate  $C'(0) = 8$ . Since the derivative is positive, the function is always increasing. Since non-negative values of  $x$  are the only ones that make sense, It is always increasing for  $x \geq 0$  and never decreasing.