Question 2: What is probability?

Probability is a number that indicates the likelihood of an occurrence happening. This number may be as low as 0 indicating the occurrence will not happen. If the probability is equal to 1, the occurrence is certain to happen. Probabilities between 0 and 1 indicate the varying levels of uncertainty about the occurrence.

A weather forecaster may indicate that there is an 80% chance of rain. This percentage may be written as 0.8. Since this number is close to 1, it is very likely that it will rain. If the forecast was for a 10% chance of rain, the probability is 0.1. A low probability like this indicates that it is not likely to rain.

Probability is defined in terms of the outcomes in the sample space of an experiment. Suppose we have an experiment with a finite number of outcomes in the sample space. Let's represent the outcomes with the letter e followed by a subscript. If there are n outcomes from the experiment, then the sample space S is

$$S = \{e_1, e_2, \ldots, e_n\}$$

The probability of each outcome is symbolized by writing $P(e_1)$, $P(e_2)$, …, $P(e_n)$. We can assign a probability to each outcome as long as the probability satisfies certain requirements.

> Each outcome of an experiment must meet two requirements.
>
> 1. The probability of each outcome is a number from 0 to 1,
>
> $$0 \le P(e_1) \le 1, \quad 0 \le P(e_2) \le 1, \ \ldots, \quad 0 \le P(e_n) \le 1$$
>
> 2. The sum of the probabilities of all outcomes is equal to 1,
>
> $$P(e_1) + P(e_2) + \cdots + P(e_n) = 1$$

We can use these requirements to determine whether the probability assignments for an experiment are valid.

Example 5 Assigning Valid Probabilities

The chief financial officer of a nanobrewery conducts an experiment to determine the likelihood that a person will order a brown ale, pale ale, or lager. The sample space for the experiment is

$$S = \{b, p, l\}$$

where the outcomes of the experiment are brown ale (b), pale ale (p), and lager (l). Determine if the probability assignments in each part are valid.

a. $P(b) = 0.5$, $P(p) = 0.4$, and $P(l) = 0.25$.

Solution The outcomes in this experiment are represented by b, p, and l instead of e_1, e_2, and e_3. However, we can still check each of the requirements listed above. Each probability is a number from 0 to 1 so the first requirement is satisfied. The sum of the probabilities is

$$P(b) + P(p) + P(l) = 0.5 + 0.4 + 0.25 \neq 1$$

Since this sum is not equal to 1, this assignment is not valid.

b. $P(b) = 0.25$, $P(p) = 0.25$, and $P(l) = 0.5$.

Solution Each probability is from 0 to 1 and the sum of the probabilities is

$$P(b) + P(p) + P(l) = 0.25 + 0.25 + 0.5 = 1$$

This is a valid probability assignment.

c. $P(b) = \frac{1}{3}$, $P(p) = \frac{1}{3}$, and $P(l) = \frac{1}{3}$.

Solution Each probability is from 0 to 1 and the sum of the probabilities is

$$P(b) + P(p) + P(l) = \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3} = 1$$

This is a valid probability assignment. In this assignment, each of the outcomes is just as likely as any other outcome. In this situation, we say the outcomes are equally likely.

A probability model for an experiment consists of the sample space for the experiment and a valid probability assignment. In Example 5, parts b and c each constitute probability models. However, the probabilities in each part are different so the models are different.

You might be inclined to ask yourself, “Which model is correct?” From a mathematical viewpoint, they are both correct since each model meets the requirements for the probabilities of the outcomes. Realistically, we might prefer one model over the other depending upon the assumptions we make about the outcomes. In the next question we’ll look at these assumptions and use them to assign probabilities.