Question 2: What is the difference between a permutation and combination?

We have looked at several ways of counting. Each counting strategy is applicable under certain assumptions. The table below outlines the assumptions and formulas for each strategy.

Strategy	Multiplication Principle	Permutations	Combinations
Purpose	Number of ways to make n choices where there are d_i ways to make i^{th} choice	Number of ways to select r objects from n different objects	
Repetition Allowed?	Yes	No	No
Order Important?	Yes	Yes	No
Formula	$d_1 \cdot d_2 \cdot \ldots \cdot d_n$	$P(n,r) = \frac{n!}{(n-r)!}$	$C(n,r) = \frac{n!}{(n-r)!\,r!}$

In many examples, more than one of these strategies may be used to count with. These examples are useful since they help to distinguish the strategies from each other. In the example below, all three examples are used.

Example 3 Corporate Board Selection

Zelbar Research has incorporated its business. The corporate bylaws require that the Board of Directors be selected from a pool nominated by shareholders. The Board consists of a chairman, vice chairman, and secretary, and five other at large board members. If the shareholders nominate 15 potential board members, how many ways are there to select the board?

Solution If we simply had to choose eight board members from the pool of 15 candidates, the number of possibilities would be $C(15,8)$.

However, part of the Board is ordered and part is not. The positions of chairman, vice chairman, and secretary are ordered. Moving people around in these positions lead to different arrangements. For the five at large board members, order does not matter. This means we need to choose the three ordered members and then the five unordered members. We'll calculate these numbers and then apply the Multiplication Principle to find the total number of ways to choose the Board Members.

Permutations are needed to choose the chairman, vice chairman, and secretary since order makes a difference. The number of ways to choose these three members from the pool of fifteen is

$$P(15,3) = \frac{15!}{(15-3)!} = 2730$$

Since order does not matter for the remaining board members, we use combinations to choose the five other members. The number of ways to select the other five members from the remaining twelve people in the pool is

$$C(12,5) = \frac{12!}{(12-5)!\,5!} = 792$$

The total number of ways to choose the Board is

$$\underbrace{2730}_{\substack{\text{Choose C,}\\ \text{VC, and}\\ \text{Sec}}} \cdot \underbrace{792}_{\substack{\text{Choose}\\ \text{at large}}} = 2{,}162{,}160$$

You might wonder if it would make a difference to choose the at large members first and the remaining three afterward. In this case, the number of ways to choose the at large members is

$$C(15,5) = \frac{15!}{(15-5)!\,5!} = 3003$$

The number of ways to choose the chairman, vice chairman, and secretary from the remaining pool is

$$P(10,3) = \frac{10!}{(10-3)!} = 720$$

If we count in this order, the Multiplication Principle gives

$$\underbrace{3003}_{\substack{\text{Choose}\\ \text{at large}}} \cdot \underbrace{720}_{\substack{\text{Choose C,}\\ \text{VC, and}\\ \text{Sec}}} = 2{,}162{,}160$$

Either counting strategy give 2,162,160 ways to select the Board of Directors.

Example 4 Choosing Stocks

The discount brokerage Sharebuilder offers a plan that allows a client to purchase up to twelve stocks per month for $12. Suppose a client wishes to invest $2000 each month in twelve different stocks.

a. How many ways are there to invest in the stocks if the client invests $500 in each of two stocks, and $100 each in the other ten stocks?

Solution There are two choice to make, choose the stocks to invest $500 in and choose the stocks to invest $100 in. Within the choices, the order does not make a difference since the client is investing the same amounts. Using the Multiplication Principle, we calculate

$$\underbrace{C(12,2)}_{\text{Choose \$500 stocks}} \cdot \underbrace{C(10,10)}_{\text{Choose \$100 stocks}} = 66 \cdot 1 = 66$$

In retrospect, we really needed to make one choice. If we had simply chosen which two stocks to invest $500 in, we would know that there is only one way to invest in the other ten stocks. However, if there were more than twelve stocks to choose from we would need to use more than one choice as shown above.

b. How many ways are there to invest in the stocks if you invest $750 in one stock, $250 in another, and $100 each in the other ten stocks?

Solution Let's break this into three choices and apply the Multiplication Principle. Start by choosing the stock to invest $750 in. Then choose the stock to invest $250 in. Finally, choose the other ten stocks. This gives

$$\underbrace{12}_{\text{Choose \$750 stock}} \cdot \underbrace{11}_{\text{Choose \$500 stock}} \cdot \underbrace{C(10,10)}_{\text{Choose \$100 stocks}} = 12 \cdot 11 \cdot 1 = 132$$

There are more choices in this case since investing different amounts for the first two stocks imposes order on those two stocks. With some foresight, we could think of the choice of the first two stocks as one choice and the other ten stocks as another choice. In this case, permutations are used make the first choice,

$$\underbrace{P(12,2)}_{\text{Choose \$750, \$500 stocks}} \cdot \underbrace{C(10,10)}_{\text{Choose \$100 stocks}} = 132 \cdot 1 = 132$$

Either strategy gives the same number of choices. The choices may give different numbers for the choices, but the product is exactly the same.