

Question 4: How do you find the likelihood of detecting a defective product?

When a product is manufactured, it is possible that the production process may lead to some products being defective. A low number of defective products is acceptable, however a high number leads to a high number of warranty claims. When the manufacturer packages the products, some of the defective products may be in the package. We can calculate the likelihood that a package will contain no defective products using combinations. The goal is to make this probability as low as possible in order to minimize the warranty costs.

Example 4 Quality Control

As the number of electronic devices increases, so does the use of rechargeable batteries. A particular manufacturer produces batteries in lots of 100. In each lot, two of the batteries will be defective. The batteries are randomly packaged in groups of four batteries. What is the probability that all of the batteries in a package will not be defective?

Solution The order in which the batteries appear in a package is irrelevant, so combinations are used to calculate the number of ways to package the batteries. The number of ways to select four batteries from the lot of 100 is

$$n(4 \text{ batteries from lot}) = C(100, 4) = 3,921,225$$

A package with no defective batteries will have batteries selected from the 98 batteries in a lot that are not defective. The number of ways to select 4 nondefective batteries from the 98 nondefective batteries is

$$n(4 \text{ nondefective batteries}) = C(98, 4) = 3,612,280$$

The probability of a package having four batteries that are not defective is

$$\begin{aligned} P(4 \text{ nondefective batteries}) &= \frac{n(4 \text{ nondefective batteries})}{n(4 \text{ batteries from lot})} \\ &= \frac{3,612,280}{3,912,225} \\ &\approx 0.923 \end{aligned}$$



If we know the likelihood of having no defective batteries in a package of four, we can also calculate the probability of the compliment of this event. The compliment of “four batteries that are not defective” is “at least one battery is defective”. This probability is

$$\begin{aligned} P(\text{At least one battery defective}) &= 1 - P(4 \text{ nondefective batteries}) \\ &\approx 1 - 0.923 \\ &\approx 0.077 \end{aligned}$$

If the cost to warranty batteries is high, this probability may be too high. Companies lower this probability by improving their manufacturing processes. This reduces the number of defective units in each lot.