Section 2.4 Counting Techniques and Probability

- How do you use the relationship between the probability of an event and its complement to simplify problems?
- How do you calculate the probability of the union and intersection of two events?
- What is conditional probability?
- How is conditional probability computed?
- How do you use probability trees to compute conditional probabilities?

How do you use the relationship between the probability of an event and its complement to simplify problems?

Key Terms

Complement

Summary

If there is a 75% chance of rain today, what are the chances it will not rain? We know that there are only two possibilities. It will either rain or it will not rain. Because the sum of the probabilities for all the outcomes in the sample space must be 100% or 1.00, we know that

$$P(\text{will rain}) + P(\text{will not rain}) = 100\%.$$

Rearranging this we see that

$$P(\text{will not rain}) = 100\% - P(\text{will rain}) = 100\% - 75\% = 25\%.$$

The events E = {will rain} and F = {will not rain} are called complements.

The **complement** of event E, denoted by E', is the set of outcomes in the sample space that are not in the event E. The probability of E' is given by

$$P(E') = 1 - P(E)$$

Notes

Guided Example 1	Practice
Many stores sell extended warranties which will allow a product to be replaced or repaired at no cost within the warranty period. If the probability that your Blue Ray player stops working before the extended warranty expires is 0.051, what is the probability that the player will not stop working before the warranty expires? **Solution** Suppose E is the event "your Blue Ray player stops working before the extended warranty expires". The compliment of this event E' is the event "your Blue Ray player does not stop working before the extended warranty expires". Using complements, $P(E') = 1 - P(E)$ $= 1 - 0.051$ $= 0.949$	For a 52-year-old male with several risk factors, the probability of developing heart disease over the next 30 years is 0.53. What is the probability of not developing heart disease over the next 30 years?

Guided Example 2	Practice
A pair of dice is rolled. What is the probability of getting a sum greater than 4? **Solution** It is possible to solve this problem without complements, but easier to use them. Because there are only six ways, (1,1), (1,2) (2,1) (2,2) (1,3) (3,1) to get a sum of 4 or less, then you can find the probability of getting 4 or less by counting these outcomes, $P(4 \text{ or less}) = \frac{6}{36}$ Using compliments, we find that the probability of getting a sum more than 4 is $P(\text{more than } 4) = 1 - \frac{6}{36} = \frac{30}{36}$ This reduces to $\frac{5}{6}$ or approximately 83%.	A pair of dice is rolled. What is the probability of getting a sum of 11 or less?

Guided Example 3	Practice
If four coins are flipped, what is the probability of obtaining at least one tail? **Solution** To begin, we make a slot or tree diagram to find the possible outcomes. There are $\underline{2}\cdot\underline{2}\cdot\underline{2}\cdot\underline{2}=16$ ways for these coins to land. Only one of those outcomes, HHHH, contains no tail. Noting that the event 'at least one tail" is the compliment of "no tails", $P(\text{at least one tail}) = 1 - P(\text{no tails})$ $= 1 - \frac{1}{16}$ $= \frac{15}{16}$ or approximately 94%.	If five coins are flipped, what is the probability of obtaining at least one head?

Guided Example 4

In 2016, the US Census Bureau estimated the number of people without medical insurance. The table below shows the ages of these uninsured people.

Age	Number Uninsured (thousands)
Under 19 years	4203
19 to 25 years	3898
26 to 34 years	6237
35 to 44 years	5252
45 to 64 years	7863
65 years and older	598

If an uninsured person is selected randomly, what is the probability that they are less than 65?

Solution This can be computed by adding all the categories less than 65, but an easier way is to compute the probability of being 65 or older and using the complement.

The total number surveyed in all categories is 28,051. From this, we can determine

$$P(\text{65 years and older}) = \frac{598}{28051}$$

The compliment of this event is calculated,

$$P(\text{less than } 65) = 1 - P(65 \text{ years and older})$$

$$= 1 - \frac{598}{28051}$$

$$= \frac{27453}{28051}$$

or about 98%

Practice

In 2016, the US Census Bureau estimated the number of people without medical insurance. The table below shows the ages of these uninsured people.

Age	Number Uninsured (thousands)
Under 19 years	4203
19 to 25 years	3898
26 to 34 years	6237
35 to 44 years	5252
45 to 64 years	7863
65 years and older	598

If an uninsured person is selected randomly, what is the probability that they are 19 or older?

How do you calculate the probability of the union and intersection of two events?

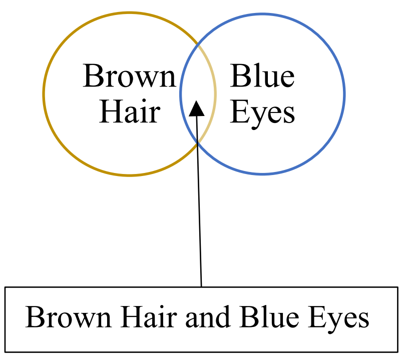
Key Terms

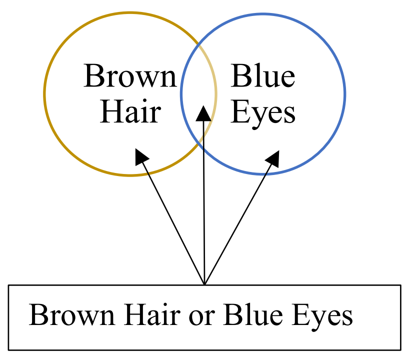
Union Intersection Mutually exclusive

Summary

Many probabilities in real life involve more than one outcome. If we draw a single card from a deck, we might want to know the probability that it is either red or a jack. If we look at a group of students, we might want to know the probability that a single student has "brown hair and blue eyes". When we combine two outcomes to make a single event, we connect the outcomes with the word "and" or the word "or." It is very important in probability to pay attention to the words "and" and "or" if they appear in a problem. The word "and" restricts the field of possible outcomes to only those outcomes that simultaneously satisfy more than one event. When we combine two events with the word "and", we call this the intersection of the events. The word "or" broadens the field of possible outcomes to those that satisfy one or both events. When we combine two events with the word "or", we call this the union of the events.

For example, students in the event "brown hair and blue eyes" must have both brown hair and blue eyes. Student in "brown hair or blue eyes" may have brown hair without blue eyes, blues eyes without brown hair, or blue eyes and brown hair.





Two sets are mutually exclusive is they have nothing in common.

Addition Rule for "Or" Probabilities

If A and B are any events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive events (they have nothing in common) then $P(A \text{ and } B) = 0$,

$$P(A \text{ or } B) = P(A) + P(B)$$

Notes

Guided Example 5	Practice
Suppose the probability of lightning on July 4 is 0.3 and the probability of rain on July 4 is 0.2, If the probability of lightning and rain on July 4 is 0.15, how likely is it that there will be rain or lightning on July 4? **Solution** Let A be the event “lightning on July 4” and B be the event “rain on July 4”. Using the formula $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ we calculate $$P(A \text{ or } B) = 0.3 + 0.2 - 0.15 = 0.35$$ The probability of lightning or rain on July 4 is 35%.	Your car won’t start in the morning. Based on his experience, your mechanic tells you • The probability you will need a new battery is 0.5. • The probability that you will need a new alternator is 0.1. • The probability that you will need a new battery and new alternator is 0.05. What is the probability that you will need a new battery or a new alternator?

<table>
<tr><th>Guided Example 6</th><th>Practice</th></tr>
<tr><td>Suppose that A and B are events and $P(A \text{ or } B) = 0.8$, $P(A) = 0.3$, and $P(B) = 0.75$. Find $P(A \text{ and } B)$.

Solution Start with the formula

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

and put in the known quantities:

$0.8 = 0.3 + 0.75 - P(A \text{ and } B)$
To solve for $P(A \text{ and } B)$,

$0.8 = 1.05 - P(A \text{ and } B)$ Combine like terms
$-0.25 = -P(A \text{ and } B)$ Subtract 1.05 from both sides
$0.25 = P(A \text{ and } B)$ Divide both sides by -1</td><td>Suppose that A and B are events and $P(A \text{ or } B) = 0.7$, $P(A) = 0.4$, and $P(A \text{ and } B) = 0.25$. Find $P(B)$.</td></tr>
</table>

<table>
<tr><th>Guided Example 7</th><th>Practice</th></tr>
<tr><td>Suppose a technology company conducts a survey of 200 consumers. Based on this survey they learn

• 23 consumers plan to purchase a laptop computer in the next six months.
• 17 consumers plan to purchase a tablet in the next six months.
• 35 consumers plan to purchase a laptop computer or tablet in the next six months

Based on this survey, what is the probability that a consumer plans to purchase a laptop computer and tablet?

Solution Let L be the event "consumer purchases a laptop computer" and T be the event "consumer purchases a tablet". In terms of these events, the information in the question leads to

$P(L) = \frac{23}{200}, P(T) = \frac{17}{200}, P(L \text{ or } T) = \frac{35}{200}$</td><td>Suppose the probability of lightning on July 11 is 0.4 and the probability of rain on July 11 is 0.35, If the probability of lightning or rain on July 11 is 0.5, how likely is it that there will be rain and lightning on July 11?</td></tr>
</table>

Now let's think of the addition rule in terms of the events L and T, $P(L \text{ or } T) = P(L) + P(T) - P(L \text{ and } T)$ Put in the probabilities and solve for $P(L \text{ and } T)$: $\frac{35}{200} = \frac{23}{200} + \frac{17}{200} - P(L \text{ and } T)$ $\frac{35}{200} = \frac{40}{200} - P(L \text{ and } T)$ $-\frac{5}{200} = -P(L \text{ and } T)$ $\frac{5}{200} = P(L \text{ and } T)$ The probability that a consumer purchases a laptop computer and a tablet in the next six months is $\frac{5}{200}$ or 2.5%.	

Guided Example 8

A wireless company surveyed 3743 customers about their gender and data usage.

Amount of Data Used	Male Users	Female Users
Less than 200 MB	650	641
200 MB up to, but not including 500 MB	442	316
500 MB up to, but not including 1 GB	291	132
1 GB up to, but not including 2 GB	172	152
2 GB or more	507	440

a. Find the likelihood that a customer will be female and use less than 200 MB of data.

Solution To find the probability that a customer will be female and use less than 200 MB of data, we need to find how many female customers used less than 200 MB and divide it by the total number of customers in the survey,

$$P(\text{female customer will use less than 200MB}) = \frac{641}{3743} \approx 0.171$$

According to the survey, the likelihood that a customer will be female and use less than 200MB of data is about 17.1%.

b. Find the probability that a customer is female.

Solution To find the probability that a customer is female, we need to find how many female customers are in the survey and divide it by the total number of customers in the survey. The total number of female customers in the survey is

$$641 + 316 + 132 + 152 + 440 = 1661$$

The likelihood of a female customer in the survey is

$$P(\text{female customer}) = \frac{1681}{3743} \approx 0.449$$

According to the survey, the likelihood that a customer is female is about 44.9%.

c. Find the probability that a customer in the survey will use less than 200 MB of data.

Solution To find the probability that a customer will use less than 200MB, we need to find how many customers use less than 200MB in the survey and divide it by the total number of customers in the survey. The total number of customers who use less than 200MB in the survey is

$$650 + 641 = 1291$$

The likelihood of a customer using less than 200MB in the survey is

$$P(\text{customer will use less than 200MB}) = \frac{1291}{3743} \approx 0.345$$

According to the survey, the likelihood that a customer uses less than 200MB of data is about 34.5%.

d. Find the probability that a customer in the survey is female or will use less than 200 MB of data.

Solution Since this question uses "or", we use the Addition Rule for "or" where A is the event "customer is female" and B is the event "customer uses less than 200MB of data"

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= \frac{1681}{3743} + \frac{1291}{3743} - \frac{641}{3743} \\ &= \frac{2331}{3743} \\ &\approx 0.623 \end{aligned}$$

The probability that a customer in the survey is female or will use less than 200 MB of data is about 62.3%

Practice

A wireless company surveyed 3743 customers about their gender and data usage.

Amount of Data Used	Male Users	Female Users
Less than 200 MB	650	641
200 MB up to, but not including 500 MB	442	316
500 MB up to, but not including 1 GB	291	132
1 GB up to, but not including 2 GB	172	152
2 GB or more	507	440

a. Find the likelihood that a user is male and will use 2 GB or more of data.

b. Find the probability that a user is male.

c. Find the probability that a user in the survey will use 2 GB or more of data.

d. Find the probability that a user in the survey is male or will use 2 GB or more of data.

What is conditional probability?

Key Terms

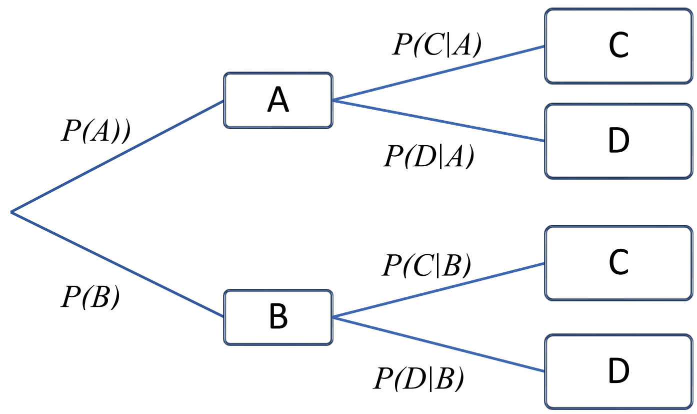
Conditional Probability Tree Diagram

Summary:

What do you think the probability is that a man is over six feet tall? If you knew that both his parents were tall would you change your estimate of the probability? A conditional probability is a probability that is based on some prior knowledge.

Conditional probability is the likelihood of an event occurring given that another event has occurred. A vertical line is used to indicate the event whose probability is being computed and the event that has already occurred. For instance, the symbols $P(A \mid B)$ correspond to the probability of A occurring given that B has already occurred. The vertical bar separates the probability we are interested in calculating from the event that is assumed to have occurred.

A tree diagram is often used to represent conditional probabilities.



In this context, the events are depicted in the boxes and the corresponding probabilities are labeled on the branches connecting the boxes. If you follow the set of branches to A and then C, note that the first branch is labeled with $P(A)$ indicating the probability of A. Continuing to C, we see that the branch is labeled $P(C \mid A)$ indicating the probability of C given that A has occurred.

<u>Notes</u>

Guided Example 9

Assume that two fair dice are rolled. Let F be the event "the total showing is seven" and E be the event "the total showing is odd".

a. Find $P(F)$.

Solution Looking at the diagram of possible dice outcomes, there are 6 ways you can get a total of 7.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Additionally, there are 36 possible rolls in the table. To find the likelihood of a total of seven, divide the number of ways to roll a total of seven by the total possible rolls,

$$P(F)=\frac{6}{36}$$

b. Find $P(F \mid E)$.

Solution If we count up the ways the total is odd, we see there are 18 ways this can happen. Of those outcomes, six of them total 7. Since we know the total is odd, the likelihood of a total of seven is

$$P(F \mid E)=\frac{6}{18}$$

Knowing that the total is odd reduces the denominator from 36 to 18 since even totals are not possible.

Practice

Assume that two fair dice are rolled. Let F be the event "the total showing is eight" and E be the event "the total showing is even".

a. Find $P(F)$.

b. Find $P(F \mid E)$.

Guided Example 10

In a poll conducted in 2016, 500 registered voters were polled with regard to their presidential preference and their party affiliation. The results are listed in the table below.

	Democrat	Republican	Other Party
Prefer Clinton	185	25	55
Prefer Trump	15	175	45

a. If a Republican voter in the poll is selected randomly, what is the likelihood that they will prefer Trump? Write your answer as a percent.

Solution In order to solve this, you need to get the totals in the Republican category so you only consider Republicans. The total number of Republicans is 25 + 175 = 200. Of those Republicans, 175 preferred Trump. So, the likelihood of preferring Trump given that you are Republican is

$$P(\text{Prefer Trump} \mid \text{Republican}) = \frac{175}{200} = \frac{7}{8}$$

or 87.5%.

b. If a voter who prefers Clinton in the poll is selected randomly, what is the probability that they are not Republican? Write your answer as a percent.

Solution We are only considering the voters who prefer Clinton, so we start with the total of that category, 185+25+55 = 265. Then total the non-Republicans, 185+55=240. The likelihood of a non-Republican given that they preferred Trump is

$$P(\text{Non-Republican} \mid \text{Prefer Clinton}) = \frac{240}{265} = \frac{48}{53}$$

or approximately 90.6%.

Practice

In a poll conducted in 2016, 500 registered voters were polled with regard to their presidential preference and their party affiliation. The results are listed in the table below.

	Democrat	Republican	Other Party
Prefer Clinton	185	25	55
Prefer Trump	15	175	45

a. If a Democrat voter in the poll is selected randomly, what is the likelihood that they will prefer Clinton? Write your answer as a percent.

b. If a voter who prefers Trump in the poll is selected randomly, what is the probability that they are not a Democrat? Write your answer as a percent.

How is conditional probability computed?

Suppose we conduct a survey in which we ask the type of operating system their phone uses and gender. The results are shown below.

	Male	Female	Total
Android	247	251	498
iOS	1201	1601	2802
Total	1448	1852	3300

We defined the events follows:

M: Consumer is male

F: Consumer is female

A: Consumer owns an Android phone

I: Consumer owns an iOS phone

To find the conditional probability $P(I \mid F)$, we recognized that we are not interested in all consumers in the survey, only the female consumers. A total of 1852 female consumers took the survey. Of those female consumers, 1601 owned a phone with the IOS operating system. The probability of a consumer owning an IOS phone given they are female is

$$P(I \mid F) = \frac{1601}{1852}$$

Let's look at these numbers more closely. The denominator is the number of female consumers in the survey. The numerator corresponds to female consumers who own an iOS phone. In words, these are consumers who are female and own an iOS phone.

Number of females and iOS phone owners

$$P(I \mid F) = \frac{1601}{1852}$$

Number of females

If we divide the top and bottom of this fraction by the number of people who took the survey,

$$P(I \mid F) = \frac{\frac{1601}{3300}}{\frac{1852}{3300}}$$

we see that the top and the bottom are now relative frequencies and can be written as

$$P(I \mid F) = \frac{P(I \text{ and } F)}{P(F)}$$

In general, we can compute conditional probability with this relationship.

Conditional Probability

If *A* and *B* are events, the likelihood of *A* occurring given that *B* has occurred is

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

provided that $P(B) \neq 0$.

Notes

<table>
<tr><th><u>Guided Example 11</u></th><th><u>Practice</u></th></tr>
<tr><td>
Suppose a batch of batteries is produced at a factory. A sample of batteries coming off the production line are sampled. From this sample, 32% of the batteries are mislabeled and 42% provide inadequate current. Twenty percent of the batteries are mislabeled and provide inadequate current.

a. What is the probability that a battery provides inadequate current given that the battery is mislabeled?

Solution Start by defining the events in the problem:

M: Battery is mislabeled
C: Battery provides inadequate current

We can match the probabilities given in problem with events:
$P(M) = 0.32$
$P(C) = 0.42$
$P(C \text{ and } M) = 0.20$

The question asks us to find the conditional probability $P(C \mid M)$. This may be found using the formula
$$P(C \mid M) = \frac{P(C \text{ and } M)}{P(M)}$$
Substitute the probabilities given in the problem to get
$$P(C \mid M) = \frac{0.20}{0.32} = 0.625$$
b. What is the probability the battery is mislabeled given that the battery provides inadequate current?

Solution To find the probability $P(M \mid C)$, apply the formula for conditional probability and substitute the values given in the problem:
</td><td>
A mathematically inclined auto mechanic determines that 35% of repairs involve dead batteries and 15% of repairs involve bad alternators. Five percent of repairs involve dead batteries and bad alternators.

a. What is the probability that a repair involves a dead battery given that the repair involves a bad alternator?

b. What is the probability the repair involves a bad alternator given that the repair involves a dead battery?
</td></tr>
</table>

$$P(M \mid C) = \frac{P(M \text{ and C})}{P(C)}$$ $$= \frac{0.20}{0.42}$$ $$\approx 0.476$$	

How do you use probability trees to compute conditional probabilities?

Key Terms

Probability tree

Summary

Let's look at the survey that was administered to a group of consumers who own a mobile phone. The results of the survey are below.

	Male	Female	Total
Android	247	251	498
iOS	1201	1601	2802
Total	1448	1852	3300

Define the events below:

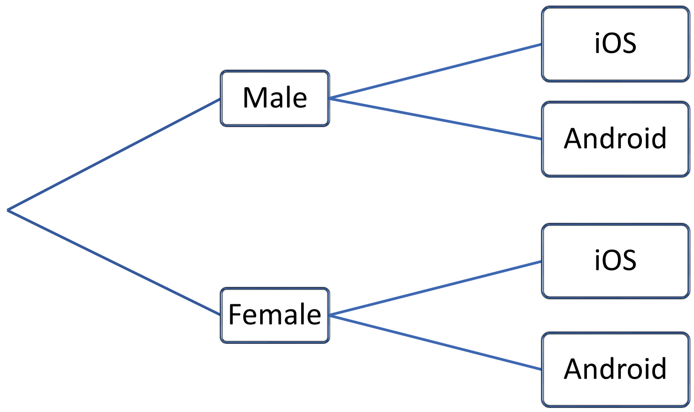
M: Consumer is male

F: Consumer is female

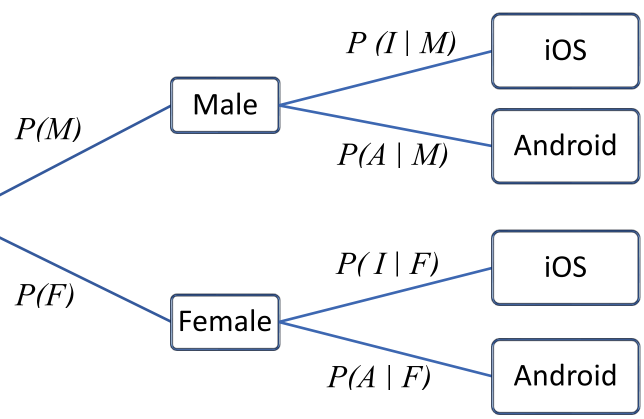
A: Consumer owns an Android phone

I: Consumer owns an iOS phone

We used this information to calculate probabilities in earlier questions. Now we want to use them to find the probabilities on each branch of the tree diagram below.



In the tree diagram, we start on the left and work to the right. The branches are labeled with the probabilities below:



Examine the diagram carefully to note that the second level of the tree consists of conditional probabilities. In each case, the given part is where the branch originates and the probability we want is where the branch terminates.

To find the first set of probabilities, calculate the relative frequencies of males and females in the survey:

$$P(M) = \frac{1448}{3300} \approx 0.439, \quad P(F) = \frac{1852}{3300} \approx 0.561$$

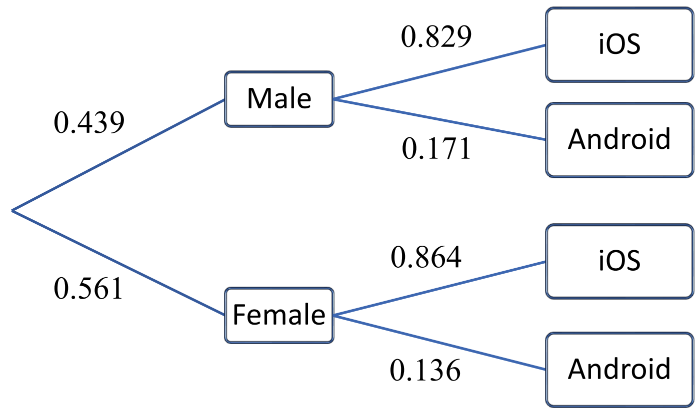
Notice that the denominator is the total number of owners in the survey.

The conditional probabilities are found by taking into account the given condition that the consumer is male, or the consumer is female:

$$P(I \mid M) = \frac{1201}{1448} \approx 0.829, \quad P(A \mid M) = \frac{247}{1448} \approx 0.171$$
$$P(I \mid F) = \frac{1601}{1852} \approx 0.864, \quad P(A \mid F) = \frac{251}{1852} \approx 0.136$$

The denominators in each of these probabilities take into account the given condition. For instance, to find $P(I \mid M)$ we divide by the total number of males instead of the total number of phone users in the survey.

Label these probabilities on the tree diagram to give



The rule for computing conditional property can be interpreted different. In the last question, we defined the conditional probability $P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$. If we multiply both side of this equation by $P(B)$, we get

$$P(A \mid B)P(B) = P(A \text{ and } B)$$

We can also apply this strategy to the conditional probability $P(B \mid A) = \dfrac{P(B \text{ and } A)}{P(A)}$ to obtain a similar expression,

$$P(B \mid A)P(A) = P(B \text{ and } A)$$

These expressions give the joint probability of A and B as a product of a conditional probability and a marginal probability.

Product Rule for Probability

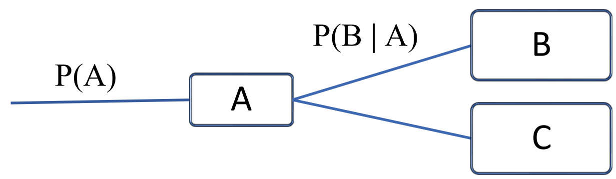
The probability of the event A and B is

$$P(A \text{ and } B) = P(A \mid B)P(B)$$

or

$$P(A \text{ and } B) = P(B \mid A)P(A)$$

We can utilize these relationships when we use a tree diagram. The probabilities on the right side of the second rule, $P(A \text{ and } B) = P(B \mid A)P(A)$, lie along the branch connecting to A followed by B. This means we can find the probability of A and B by multiplying the probabilities that connect to A followed by B.



Product Rule for Tree Diagrams

The product of all probabilities along a branch on a tree diagram is the likelihood of all events occurring that are on the branch.

<u>Notes</u>

Guided Example 12

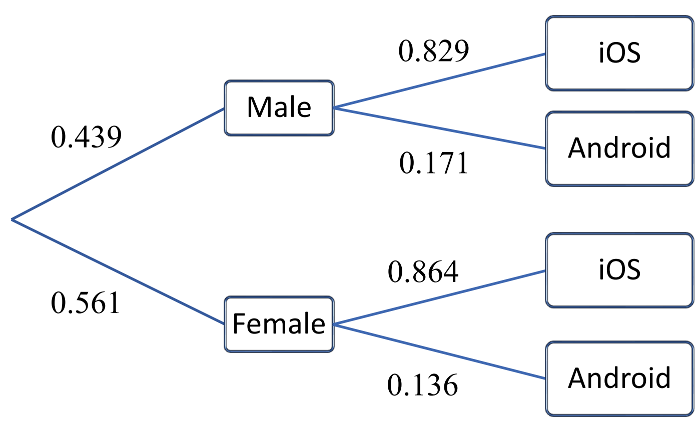
A survey is administered to a group of consumers who own a smartphone. The results of the survey are shown below.

	Male	Female	Total
Android	247	251	498
iOS	1201	1601	2802
Total	1448	1852	3300

Define the events below:

M: Consumer is male
F: Consumer is female
A: Consumer owns an Android phone
I: Consumer owns an iOS phone

In an earlier example, we used these data and events to create the tree diagram below.



Compute the likelihood that a consumer is male and owns an iphone.

Solution In terms of the events, we are being asked to find $P(M \text{ and } I)$. Apply the formula for finding intersections of events to give

$$P(M \text{ and } I) = P(I \mid M)P(M)$$

The probabilities on the right side are found along the branch through Male and iOS. Put these into the formula to yield

$$P(M \text{ and } I) = (0.829)(0.439) \approx 0.364$$

<u>Practice</u>

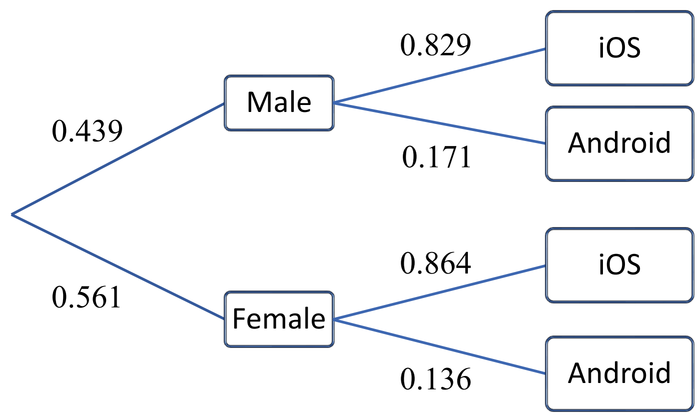
A survey is administered to a group of consumers who own a smartphone. The results of the survey are shown below.

	Male	Female	Total
Android	247	251	498
iOS	1201	1601	2802
Total	1448	1852	3300

Define the events below:

M: Consumer is male
F: Consumer is female
A: Consumer owns an Android phone
I: Consumer owns an iOS phone

In an earlier example, we used these data and events to create the tree diagram below.



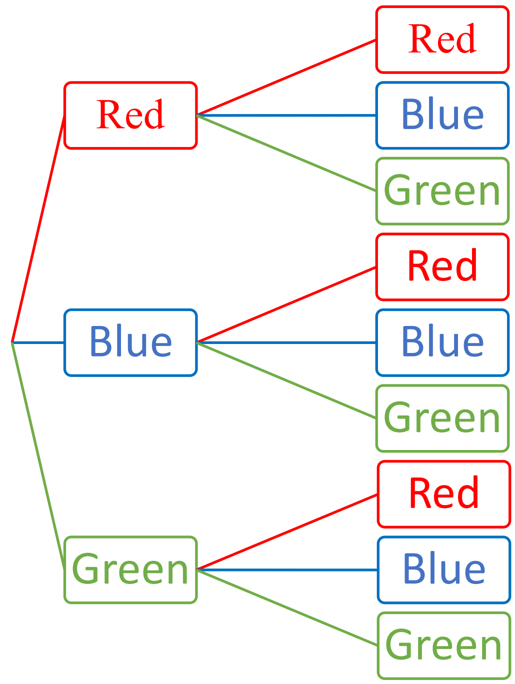
Compute the likelihood that a consumer is female and owns an Android phone.

Guided Example 13

Suppose you have a jar containing 10 red balls, 5 blue balls and 15 green balls.

a. If two balls are selected from the jar without replacement, what is the probability that the first ball is red, and the second ball is red?

Solution We can find the probability that the first ball is red, and the second ball is red by labeling the appropriate branches of a tree diagram.



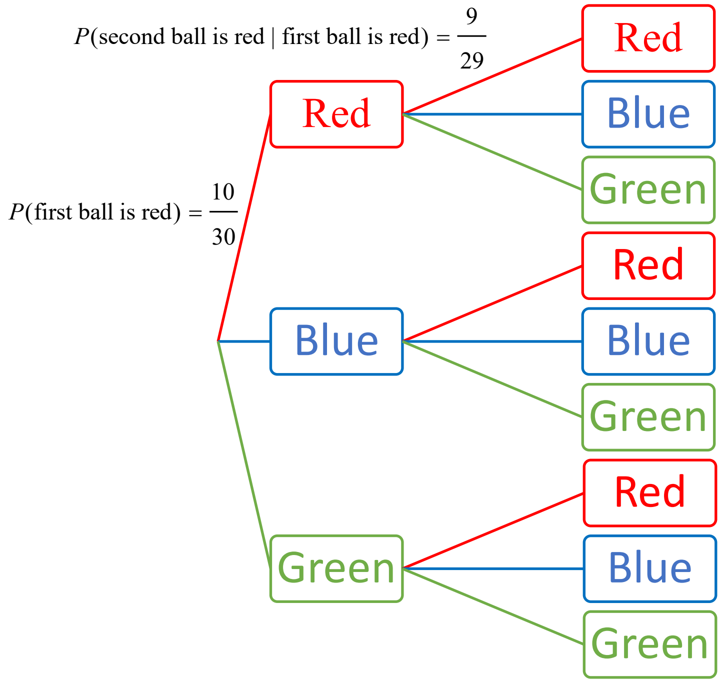
Since there are 30 balls in the jar and 10 of them are red,

$$P(\text{first ball is red}) = \frac{10}{30}$$

Once the first ball is selected, only 29 balls remain and nine of those balls are red. The probability that the second ball is red given that the first ball is red is

$$P(\text{second ball is red} \mid \text{first ball is red}) = \frac{9}{29}$$

We can now label the branch through first ball red and second ball red:



Since the events we are interested in concern this branch only, there is no need to label any other branches. The probability that the first ball is red, and the second ball is red is the product of these probabilities,

$$P(\text{first ball is red and second ball is red}) = \frac{10}{30} \cdot \frac{9}{29} \approx 0.103$$

or about 10.3%.

b. If two balls are selected from the jar without replacement, what is the probability of selecting a red ball and a blue ball?

Solution We could follow the strategy from the previous part and draw the tree diagram or we can simply visualize branches passing through red and blue balls. There are two such branches since the first ball could be red and the second ball blue or the first ball could be blue and the second ball red.

In the first case, the first segment would be labeled

$$P(\text{first ball is red}) = \frac{10}{30}$$

Once the first ball is chosen, there are 29 balls remaining. Of those remaining 29 balls, 5 are blue telling us

$$P(\text{second ball is blue} \mid \text{first ball is red}) = \frac{5}{29}$$

The probability that the first ball is red, and the second ball is blue is the product of the probabilities along the branch,

$$P(\text{first ball is red and second ball is blue}) = \frac{10}{30} \cdot \frac{5}{29} \approx 0.057$$

or about 5.7%.

In the second case, the first segment would be labeled

$$P(\text{first ball is blue}) = \frac{5}{30}$$

Once the first ball is chosen, there are 29 balls remaining. Of those remaining 29 balls, 10 are red telling us

$$P(\text{second ball is red} \mid \text{first ball is blue}) = \frac{10}{29}$$

The probability that the first ball is blue, and the second ball is red is the product of the probabilities along the branch,

$$P(\text{first ball is blue and second ball is red}) = \frac{5}{30} \cdot \frac{10}{29} \approx 0.057$$

or about 5.7%

Either one of these cases satisfies selecting a red and a blue ball so the total probability is the sum of these cases, 5.7% + 5.7% = 11.4%.

Practice

Suppose you have a jar containing 10 red balls, 5 blue balls and 15 green balls.

a. If two balls are selected from the jar without replacement, what is the probability that the first ball is green, and the second ball is red?

b. If two balls are selected from the jar without replacement, what is the probability of selecting a green ball and a red ball?

<u>Guided Example 14</u>

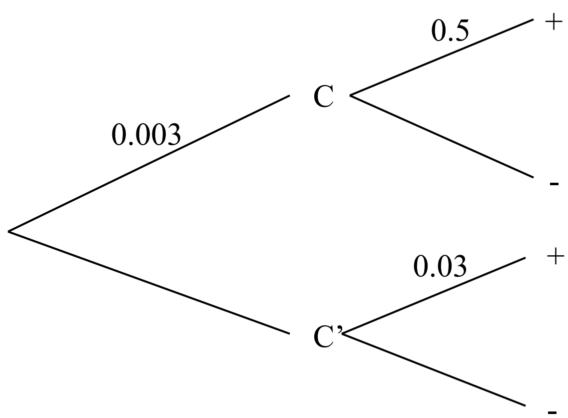
The probability of colorectal cancer can be given as 0.3%. If a person has colorectal cancer, the probability that the hemoccult test is positive is 50%. If a person does not have colorectal cancer, the probability that he still tests positive is 3%. What is the probability that a person has colorectal cancer and tests negative?

Solution To solve this problem, we'll draw and label an appropriate tree diagram.

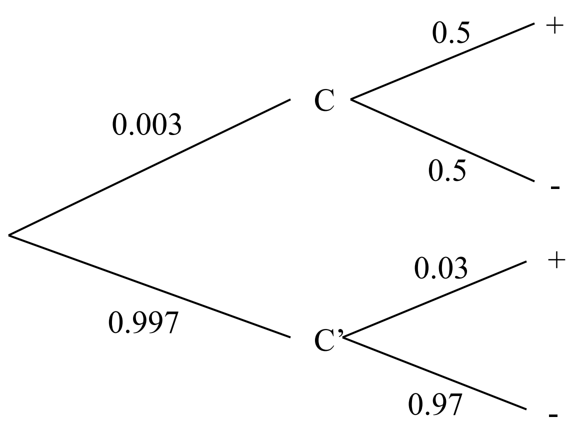
Look at the information given in the problem. If C is the event "person has colorectal cancer" , C' is the event "person does not have colorectal cancer, + is the event "the hemoccult test is positive" and – is the event "the hemoccult test is negative", we know that

$$P(C) = 0.003 \quad P(+\,|\,C) = 0.5 \quad P(+\,|\,C') = 0.03$$

This suggests the following tree diagram:



Knowing that the sum of an event and its compliment should add to 1, we can finish the tree diagram as follows:



The probability we are looking for is $P(C \text{ and -})$. This probability is the product of the probabilities along the segments through C and -,

$$P(C \text{ and -}) = 0.003 \cdot 0.5 = 0.0015$$

or 0.15%.

Practice

The probability of colorectal cancer can be given as .3%. If a person has colorectal cancer, the probability that the hemoccult test is positive is 50%. If a person does not have colorectal cancer, the probability that he still tests positive is 3%. What is the probability that a person does not have colorectal cancer and tests positive?