

Section 3.2 Dimensional Analysis

- What is dimensional analysis?

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Key Terms

Dimensional analysis

Summary

Until now we have worked primarily in one measurement system or the other. Now we will convert between the US Customary System and the Metric system. To do this, we will use dimensional analysis and unit fractions.

Here are basic conversion factors for each type of measurement:

Length

1 foot (ft) = 12 inches (in)

1 yard (yd) = 3 feet (ft)

1 mile = 5,280 feet

1 meter \approx 3.281 feet

1 inch \approx 2.54 centimeters

Weight and Mass

1 pound (lb) = 16 ounces (oz)

1 ton = 2000 pounds

1 kilogram \approx 2.2 pounds

Volume

1 cup = 8 fluid ounces (fl oz)*

1 pint = 2 cups

1 quart = 2 pints = 4 cups

1 gallon = 4 quarts = 16 cups

1 gallon \approx 3.785 liters

1 liter \approx 33.8 fluid ounces

*Fluid ounces are a capacity measurement for liquids. 1 fluid ounce \approx 1 ounce (weight) for water only.

Unit fractions are fractions which help us to convert from one unit to another. For instance, suppose we want to convert 48 inches to feet. From the table above, we see that 1 ft = 12 in. We can convert this relationship to two unit fractions,

$$\frac{1 \text{ ft}}{12 \text{ in}} \text{ and } \frac{12 \text{ in}}{1 \text{ ft}}$$

In each of these unit fractions, the distance in the top and bottom are physically equal. When the quantities in the top and bottom of a fraction are equal, the fraction is equal to one. However, these fractions do not “look” equal to one since our eyes are drawn to the numbers and not the units. The units on each number are what make the fraction equal to one.

To make the conversion from 48 inches to feet, we multiply by the appropriate unit fraction, $\frac{1 \text{ ft}}{12 \text{ in}}$. This allows us to treat the units like numbers and cancel units:

$$\begin{aligned} \frac{48 \text{ in}}{1} \cdot \frac{1 \text{ ft}}{12 \text{ in}} &= \frac{48 \cancel{\text{ in}} \cdot 1 \text{ ft}}{1 \cdot 12 \cancel{\text{ in}}} \\ &= \frac{48 \text{ ft}}{12} \\ &= 4 \text{ ft} \end{aligned}$$

If you are familiar with US/Customary units, this is not very surprising. So, let's look at a more complicated example where we convert 100 meters per second to miles per hour.

Start by writing the rate 100 feet per second as a fraction:

$$\frac{100 \text{ ft}}{1 \text{ sec}}$$

The “per” indicates division. This fraction indicates a distance of 100 feet covered in a time of 1 second. To convert to miles per hour, we must change feet to miles and seconds to hours. We'll do this with several unit fractions:

$$\frac{1 \text{ mi}}{5280 \text{ ft}}, \frac{60 \text{ sec}}{1 \text{ min}}, \text{ and } \frac{60 \text{ min}}{1 \text{ hr}}$$

Multiply these unit fractions times 100 feet per second:

$$\begin{aligned} \frac{100 \cancel{\text{ ft}}}{1 \cancel{\text{ sec}}} \cdot \frac{1 \text{ mi}}{5280 \cancel{\text{ ft}}} \cdot \frac{60 \cancel{\text{ sec}}}{1 \cancel{\text{ min}}} \cdot \frac{60 \cancel{\text{ min}}}{1 \text{ hr}} &= \frac{100 \cdot 1 \cdot 60 \cdot 60 \text{ mi}}{1 \cdot 5280 \cdot 1 \cdot 1 \text{ hr}} \\ &\approx 68.2 \frac{\text{mi}}{\text{hr}} \end{aligned}$$

A speed of 100 feet per second is approximately equal to 68.2 miles per hour.

NotesGuided Example 1

Convert 2.2 miles to feet.

Solution We know that 1 mile is equal to 5280 feet. Using this in a unit fraction and multiplying gives

$$\frac{2.2 \cancel{\text{mi}}}{1} \cdot \frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} = 11616 \text{ ft}$$

Practice

Convert 10 yards to feet

Guided Example 2Practice

Convert 13 feet to centimeters.

Solution We know two facts: $2.54 \text{ cm} \approx 1 \text{ inch}$ and $12 \text{ in} = 1 \text{ ft}$. Combine these facts in unit fractions and multiply:

$$\frac{13 \cancel{\text{ft}}}{1} \cdot \frac{12 \cancel{\text{in}}}{1 \cancel{\text{ft}}} \cdot \frac{2.54 \text{ cm}}{1 \cancel{\text{in}}} = \frac{13 \cdot 12 \cdot 2.54 \text{ cm}}{1} = 396.24 \text{ cm}$$

Convert 200 fluid ounces to liters.

Guided Example 3

The velocity of a 50-caliber bullet as it leaves the barrel of a gun is 853 meters per second.

What is the velocity in miles per hour?

Solution Several fact will help us to make this conversion:

1 meter \approx 3.281 feet
 5280 feet = 1 mile
 60 seconds = 1 minute
 60 minutes = 1 hour

Use these in unit fraction to make the conversion:

$$\frac{853 \cancel{\text{m}}}{1 \cancel{\text{sec}}} \cdot \frac{3.281 \cancel{\text{ft}}}{1 \cancel{\text{m}}} \cdot \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} \cdot \frac{60 \cancel{\text{sec}}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} = \frac{853 \cdot 3.281 \cdot 1 \cdot 60 \cdot 60 \text{ mi}}{1 \cdot 1 \cdot 5280 \cdot 1 \cdot 1 \text{ hr}} = \frac{10075294.8 \text{ mi}}{5280 \text{ hr}} \approx 1908.2 \frac{\text{mi}}{\text{hr}}$$

So, 853 meters per second is approximately 1908.2 miles per hour.

Practice

The mileage of a new car is 48 miles per gallon.

What is this mileage in kilometers per liter?