Section 4.2 Modeling with Linear Equations

- How can you use a point and slope to build a linear model?
- How can you use two points to build a linear model?

How can you use a point and slope to build a linear model?

Key Terms

Slope Slope-Intercept Form

Summary

The **slope of a line represents the rate of change of that line**, that is, the rate at which the dependent variable (y) is increasing or decreasing as the independent variable (x) increases. An increasing quantity is represented by a positive slope, and a decreasing quantity is represented by a negative slope.

The equation of a line can be written in several forms, but probably the most useful and common form is the slope-intercept form:

> **Slope-Intercept Form**: If a line has a slope m, and a y-intercept b, then its equation in slope-intercept form is
>
> $$y = mx + b$$
>
> It is often useful when working with a linear model in this form to think of m as the rate of change of the quantity, and b as the fixed or initial value of the quantity.

Recall in a previous example we used the model $y = -\frac{1}{2}x + 5$ where y was the number of acres left to mow after x hours. Notice this equation is in the form $y = mx + b$ where m is $-\frac{1}{2}$ and b is 5. This means there were initially 5 acres to mow, and the number of acres left to mow was decreasing at a rate of half an acre per hour.

Given a general point and the slope, you must first solve for the y-intercept, b, before you can write the equation in slope-intercept form. To do this, you can use the x-coordinate, y-coordinate, and slope m to solve for b as illustrated below.

Notes

Guided Example 1	Practice
Find a linear equation of the line with slope 2 passing through the point (-1,4). **Solution** Given a slope of 2, an x-coordinate of -1 and a y-coordinate of 4, we can substitute these values into the slope-intercept form $y = mx + b$ and solve for b. $4 = 2(-1) + b$ — Substitute values for x, y, and m $4 = -2 + b$ — Simplify $6 = b$ — Add 2 to both sides Now that we have m and b we can write the equation in slope-intercept form: $y = 2x + 6$.	Find a linear equation of the line with slope -5 passing through the point (1,1).

Guided Example 2	Practice
In 2017 a company reported a revenue of \$20.15 million, and it had increased at a rate of \$1.2 million per year. Use this information to create a linear model and predict the company's revenue in 2025. **Solution** We will define x to be the number of years after 2017, and y to be the company's revenue in millions of dollars. Since the revenue was \$20.15 million in the year 2017, we can write that as the point (0, 20.15). The revenue is increasing by a rate of \$1.2 million per year so the slope m is 1.2. Notice by defining x in this way, 20.15 is our initial value or y-intercept, so we already know that b is 20.15. Now we write our equation in slope-intercept form: $y = 1.2x + 20.15$ To predict the revenue is 2025, first we need to note that 2025 is 8 years after 2017, so we will	In 2016 the average student loan debt per borrower was \$32,731 and was increasing by \$6,546.20 each year. Use this information to create a linear model and predict the year when the average debt will reach \$50,000.

substitute 8 into the equation for x, and solve for y: $y = 1.2(8) + 20.15$ Substitute 8 for x $y = 9.6 + 20.15$ Simplify $y = 29.75$ According to our model, in the year 2025 the company's revenue will be \$29.75 million.	

How can you use two points to build a linear model?

Key Terms

Slope Formula

Summary

Instead of being given a point and a slope, sometimes we are only given two points with which to construct our model. Our first task in that situation is to calculate the slope of the line that passes through those points. On the *xy*-plane, we define slope as the change in y over the change in x, and we get the following formula:

> **Slope Formula**: If a line passes through two points (x_1, y_1) and (x_2, y_2) we can calculate the slope, m, of that line using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Given two points, we can use the slope formula to find the rate of change of the quantity. Then we can proceed in constructing our model by using a point and the slope to solve for b, or we can define x in such a way as to use a point as our initial value, as seen in the previous exercises.

Notes

Guided Example 3	Practice
Find the equation of the line passing through the points (2, 4) and (-1, 5). **Solution** We will use the point (2,4) as (x_1, y_1) and (-1, 5) as (x_2, y_2) and substitute into the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$: $m = \frac{5-4}{-1-2}$ $= \frac{1}{-3}$ $= -\frac{1}{3}$ Next, we can choose either point and use the slope to solve for *b* as we did in the previous examples. We will use the first point (2, 4) for *x* and *y* and $-\frac{1}{3}$ for the slope *m* in the slope-intercept form $y = mx + b$: $4 = -\frac{1}{3}(2) + b$ Substitute values for x, y, and m $4 = -\frac{2}{3} + b$ Simplify $\frac{14}{3} = b$ Add $\frac{2}{3}$ to both sides The equation in slope-intercept form is: $y = -\frac{1}{3}x + \frac{14}{3}$.	Find the equation of the line passing through the points (-3,1) and (6, -7).

Guided Example 4	Practice
Suppose the average starting salary for a bachelor's degree recipient in the US was \$45,473 in 2014, and in 2018 it increased to \$50,390. Model this information with a linear equation and predict the year when the average starting salary will reach \$65,000. **Solution** We are given two salaries with their corresponding years, and if we define x as the number of years after 2014 and y as the salaries, then we can write \$45,473 in 2014 as the point (0, 45473) and \$50,390 in 2018 as the point (4, 50,390). Next, we use these two points in the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$: $m = \frac{50390 - 45473}{4 - 0} = \frac{4917}{4} \approx 1229$ Our slope is 1,229 and the y-intercept is 45,473. This gives us the model $y = 1229x + 45473$. To predict the year when the average salary will reach \$65,000, we will substitute 65,000 into the equation for y and solve for x: $65000 = 1229x + 45473$ Substitute 65000 for x $19527 = 1229x$ Subtract 45473 from both sides $16 \approx x$ Divide both sides by 1229 Since we defined x as the number of years 2014, an x value of 16 corresponds to the year 2030. So according to our model, the average salary will reach \$65,000 in approximately the year 2030.	Suppose there were 117 thousand computer programming jobs in 2010 and that number increased to 295 thousand in 2017. Model this information with a linear equation and predict the year the number of computer programming jobs will reach 500,000.