

Suppose a loan of \$2500 is made to an individual at 6% interest compounded quarterly. The loan is repaid in 6 quarterly payments.

- Find the payment necessary to amortize each loan.
- Find the total payments and the total amount of interest paid based on the calculated monthly payments.
- Find the total payments and the total amount of interest paid based on an amortization table.

- a. To find the payment on the loan, use the formula

$$PMT = \frac{i \cdot PV}{1 - (1 + i)^{-n}}$$

For this problem, the interest rate per period is  $i = \frac{0.06}{4}$ . The present value is  $PV = 2500$  and the number of periods is  $n = 6$ . Using these values gives

$$PMT = \frac{\frac{0.06}{4} \cdot 2500}{1 - \left(1 + \frac{0.06}{4}\right)^{-6}} \approx 438.813$$

Depending on how the rounding is done, this gives a payment of \$438.81 or 438.82. For a calculated payment, the payment is often rounded to the nearest penny. However, many finance companies will round up to insure the last payment is no more than the other payments.

- b. The total payments (assuming the payment is rounded to the nearest penny) is

$$438.81(6) = 2632.86$$

so the total amount of interest is

$$2632.86 - 2500 = 132.86$$

- c. Making the amortization takes several steps. Let me take it in pieces using the payment from above.

Payment Number	Amount of Payment	Interest in Payment	Principal in Payment	Outstanding Principal after Payment
0				2500
1	438.81	37.50	401.31	2098.69

$\frac{0.06}{4} \cdot 2500$

$438.81 - 37.5$

$2500 - 401.31$

The next row is filled out in a similar manner.

Payment Number	Amount of Payment	Interest in Payment	Principal in Payment	Outstanding Principal after Payment
0				2500
1	438.81	37.50	401.31	2098.69
2	438.81	31.48	407.33	1691.36

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \boxed{\frac{0.06}{4} \cdot 2098.69} & \boxed{438.81 - 31.48} & \boxed{2098.69 - 407.33} \end{array}$$

Continue this process until the last row

Payment Number	Amount of Payment	Interest in Payment	Principal in Payment	Outstanding Principal after Payment
0				2500
1	438.81	37.50	401.31	2098.69
2	438.81	31.48	407.33	1691.36
3	438.81	25.37	413.44	1277.92
4	438.81	19.17	419.64	850.28
5	438.81	12.87	425.94	432.34
6				

After the fifth payment, we have \$432.34 of principal left to pay in the final payment. So this is the principal in the sixth payment. The interest is found by paying interest on the outstanding balance,

$$\frac{0.06}{4} \cdot 432.34 \approx 6.49$$

This gives a final payment of

$$432.34 + 6.49 = 438.83$$

Now put these numbers into the amortization table.

Payment Number	Amount of Payment	Interest in Payment	Principal in Payment	Outstanding Principal after Payment
0				2500
1	438.81	37.50	401.31	2098.69
2	438.81	31.48	407.33	1691.36
3	438.81	25.37	413.44	1277.92
4	438.81	19.17	419.64	850.28
5	438.81	12.87	425.94	432.34
6	438.83	6.49	432.34	0

Since the payments had been rounded to the nearest penny (rounded down), the final payment is slightly higher than the previous payments.