Most antiderivatives can be done with a little algebra and couple basic rules. Like derivatives, you can take the antiderivatives of a sum or difference by finding the antiderivative of the pieces and summing or subtracting the pieces. Constants can also be pulled through antiderivatives just like they can be in derivatives. Here are the most basic antiderivatives:

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ Power Rule
2. $\int e^x dx = e^x + C$
3. $\int a^x dx = \frac{a^x}{\ln(a)} + C$
4. $\int \frac{1}{x} dx = \ln|x| + C$

All of the problems in 15.1 can be done with a little algebra grunt work and one of these rules. Let's look at some examples.

Find the antiderivative of $x\left(x^2-3\right)$ with respect to x.

This problem is essentially asking us to do $\int x\left(x^2-3\right)dx$. To do this, we'll need to use the power rule. The key is to make the product look like a sum:

$$\int x\left(x^2-3\right)dx = \int\left(x^3-3x\right)dx$$ Multiply the product out

$$= \int x^3 dx - 3\int x dx$$ Write the difference as the difference of the antiderivatives

$$= \frac{x^4}{4} - 3\frac{x^2}{2} + C$$

$$= \frac{x^4}{4} - \frac{3x^2}{2} + C$$

Take the antiderivative with the power rule

Find $\int \left(15x\sqrt{x} + 2\sqrt{x}\right) dx$

Start by rewriting the square roots as ½ powers. Then use algebra to transform the integrand so we can use the power rule.

$$\int \left(15x\sqrt{x} + 2\sqrt{x}\right) dx = \int \left(15x \cdot x^{1/2} + 2x^{1/2}\right) dx$$

$$= \int \left(15x^{3/2} + 2x^{1/2}\right) dx$$ Add the exponents when you multiply

$$= 15\int x^{3/2} dx + 2\int x^{1/2} dx$$ Break up the sum into individual antiderivatives

$$= 15 \cdot \frac{x^{5/2}}{5/2} + 2 \cdot \frac{x^{3/2}}{3/2} + C$$

Apply the power rule and simplify the result

$$= 6x^{5/2} + \tfrac{4}{3}x^{3/2} + C$$

$$= 6\sqrt{x^5} + \tfrac{4}{3}\sqrt{x^3} + C$$

In the last step the fractional powers have been rewritten as roots using the rule $x^{m/n} = \sqrt[n]{x^m}$.

Find $\int \frac{1}{3x^2} dx$

Start by pulling the constant out of the antiderivative. Then you can apply the power rule.

$$\int \frac{1}{3x^2} dx = \frac{1}{3}\int \frac{1}{x^2} dx$$

Pull out the constant in the denominator and rewrite with a negative power.

$$= \frac{1}{3}\int x^{-2} dx$$

$$= \frac{1}{3} \cdot \frac{x^{-1}}{-1} + C$$ Take the antiderivative with the power rule

$$= -\frac{1}{3} \cdot \frac{1}{x} + C$$

Rewrite without a negative exponent.

$$= -\frac{1}{3x} + C$$

Find $\int \frac{1+2t^3}{t}\,dt$

The key to this problem is to divide the denominator into both terms on top. Then you can apply the rules.

$$\int \frac{1+2t^3}{t}\,dt = \int \left(\frac{1}{t}+\frac{2t^3}{t}\right)dt$$

Break up the antiderivative

$$= \int \frac{1}{t}\,dt + \int \frac{2t^3}{t}\,dt$$

Simplify the fractions

$$= \int \frac{1}{t}\,dt + 2\int t^2\,dt$$

Apply the antiderivative rules

$$= \ln|t| + 2\cdot\frac{t^3}{3} + C$$