Section 10.5 Continuous Functions

Question 1 – What does a continuous function look like?

Question 2 – How is a limit related to a continuous function?

Question 1 – What does a continuous function look like?

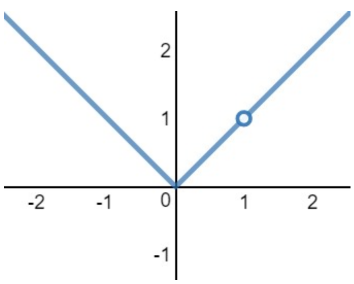
<u>Key Terms</u>

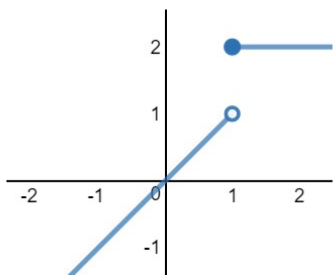
Continuous at a point   Discontinuity

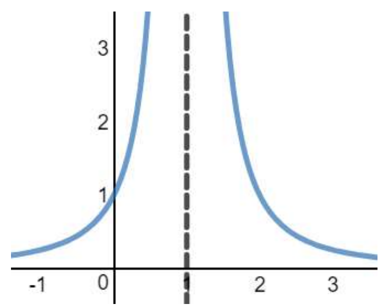
Continuous function

<u>Summary</u>

In a graphical sense, a continuous function is a function that can be drawn without lifting you writing utensil off the paper. Discontinuities are where you must lift your writing utensil up. Polynomials, such as lines and parabolas, are continuous everywhere. Piecewise functions may or may not be continuous even if the pieces themselves are polynomials.

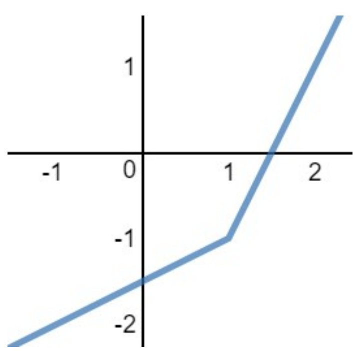






Each of the functions above Have a discontinuity at $x = 1$. The graph on the left has a hole because it is not defined there. The graph in the middle has a gap at $x = 1$. The graph on the right has a vertical asymptote at $x = 1$ that means it is not defined at $x = 1$.

Kinks in graphs do not cause them to be discontinuous by themselves.



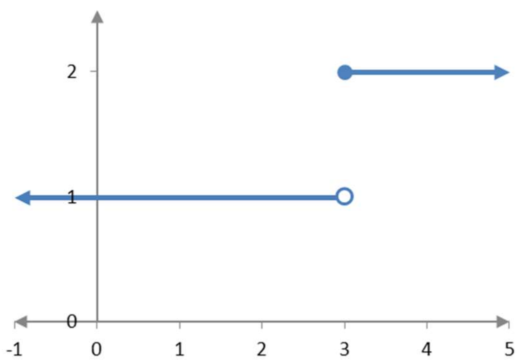
The graph above is continuous everywhere despite the kink in its appearance.

Notes

Guided Example

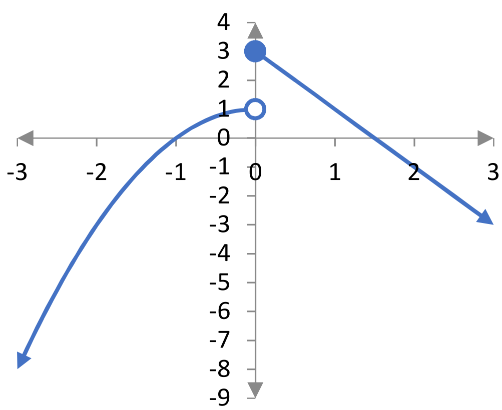
Where is the function not continuous?

a.



Solution To be continuous, you should be able to draw the function without lifting your drawing utensil. In the case of this function, you would need to lift your utensil at $x = 3$ to jump to the other piece. Because of this, the function is discontinuous at $x = 3$.

b.

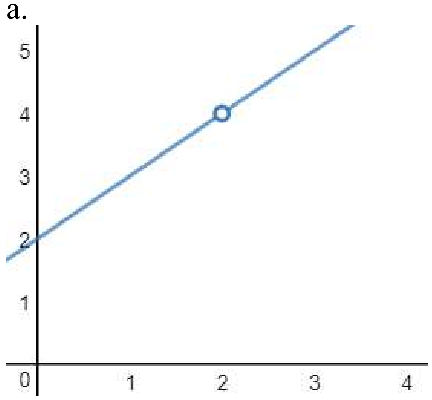


Solution To draw this function, you would need to lift your drawing utensil at $x = 0$ to jump to the other piece of the function, Therefore, the function is discontinuous at $x = 0$.

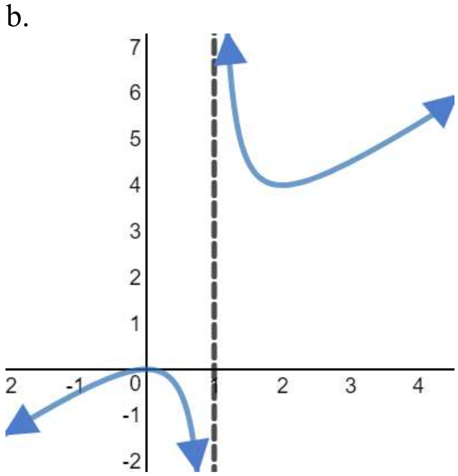
Practice

1. Where is the function not continuous?

a.



b.



Question 2 – How is a limit related to a continuous function?

Key Terms

Continuous at a point Discontinuity

Continuous function

Summary

The definition of a continuous function is related to limits. In order for a function $f(x)$to be continuous at some point x = a, three conditions must be met.

1. $f(a)$ is defined.
2. $\lim_{x \to a} f(x)$ exists.
3. $\lim_{x \to a} f(x) = f(a)$

To utilize this definition, you must compute the limits from the left and right (to make sure the limit exists) and compare those values to $f(a)$. If the left-hand limit, the right-hand limit, and $f(a)$ are all the same, then the function is continuous at $x = a$.

Notes

Guided Example	Practice
Compute the limits to determine whether the function $$f(x)=\begin{cases}-x+4 & \text{if } x<3\\ x-2 & \text{if } x>3\\ 0 & \text{if } x=3\end{cases}$$ to determine where $f(x)$ is continuous at $x=3$.	1. Compute the limits to determine whether the function $$f(x)=\begin{cases}-x^2+1 & \text{if } x<0\\ -2x+3 & \text{if } x\geq 0\end{cases}$$ to determine where $f(x)$ is continuous at $x=0$.
a. $\lim\limits_{x\to 3^-} f(x)$ **Solution** To do the left-hand limit, we use the portion of the graph that is valid for x < 3 and put that in place of $f(x)$. Then we can evaluate the limit by setting x = 3 in that formula. $$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-} -x+4 \\ =1$$	a. $\lim\limits_{x\to 0^-} f(x)$
b. $\lim\limits_{x\to 3^+} f(x)$ **Solution** For the right-hand limit, use the portion of the formula valid for $x > 3$ and put in the point: $$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+} x-2 \\ =1$$	b. $\lim\limits_{x\to 0^+} f(x)$
c. Is the function continuous at $x=3$? **Solution** For the function to be continuous at x = 3, the left-hand limit, the right-hand limit, and $f(3)$ must be equal. Since they are not, the function is discontinuous at x = 3.	c. Is the function continuous at $x=0$?

Guided Example	Practice
A state in the Midwest has a graduated state income tax given by $T(x)=\begin{cases}0.01x & \text{if } 0\le x\le 10{,}000\\ 0.015x+b & \text{if } x>10{,}000\end{cases}$ where x is the taxable income of an individual taxpayer and b is a constant. a. $\lim_{x\to 10{,}000^-} T(x)$ **Solution** To evaluate the left-hand limit, use the top portion of the piecewise function that is valid for $0\le x\le 10{,}000$: $\lim_{x\to 10{,}000^-} T(x)=\lim_{x\to 10{,}000^-} 0.01x=0.01(10{,}000)=100$ b. $\lim_{x\to 10{,}000^+} T(x)$ **Solution** For the right-hand limit, use the portion of the piecewise function that is valid for $x > 10{,}000$: $\lim_{x\to 10{,}000^+} T(x)=\lim_{x\to 10{,}000^+} 0.015x+b$ $=0.015(10{,}000)+b$ $=150+b$ c. What value of b would make this function continuous? **Solution** For the function to be continuous, the left and right-hand limits must be equal. Set the limits equal and solve for b: $100=150+b$ $-50=b$ Also note that $f(10{,}000)=0.01(10{,}000)=100$.	2. A state in the East has a graduated state income tax given by $T(x)=\begin{cases}0.03x & \text{if } 0\le x\le 15{,}000\\ 0.045x+b & \text{if } x>15{,}000\end{cases}$ where x is the taxable income of an individual taxpayer and b is a constant. a. $\lim_{x\to 15{,}000^-} T(x)$ b. $\lim_{x\to 15{,}000^+} T(x)$ c. What value of b would make this function continuous?