Evaluate the difference quotient $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 - x$.

This is a little different from $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, but works the same way. Since a value is not supplied for x, we just leave it and work out the limit. Start by evaluating $f(x+h)$:

$$\begin{aligned} f(x+h) &= (x+h)^2 - (x+h) \\ &= x^2 + 2xh + h^2 - x - h \end{aligned}$$

Make sure you FOIL the square out and distribute the negative.

Now put this along with $f(x)$ into the difference quotient.

$$\begin{aligned} \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{\left(x^2 + 2xh + h^2 - x - h\right) - \left(x^2 - x\right)}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2 - h}{h} \\ &= \lim_{h \to 0} \frac{h(2x + h - 1)}{h} \\ &= \lim_{h \to 0} (2x + h - 1) \end{aligned}$$

As h gets smaller and smaller, the term in the middle gets smaller. This means the limit is equal to $2x - 1$. Since the other terms do not contain x, they are unaffected when h gets small.