Evaluate the difference quotient $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for $f(x)=\frac{1}{x}$.

This is a little different from $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, but works the same way. Since a value is not supplied for x, we just leave it and work out the limit. Start by evaluating $f(x+h)$:

$$f(x+h)=\frac{1}{x+h}$$

Now put this along with $f(x)$ into the difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h}$$

Get a common denominator of $x(x+h)$

$$=\lim_{h\to 0}\frac{\frac{-h}{x(x+h)}}{h}$$

Combine the fractions in the numerator together

$$=\lim_{h\to 0}\frac{-h}{x(x+h)}\frac{1}{h}$$

Dividing by h is the same as multiplying by its reciprocal

$$=\lim_{h\to 0}\frac{-1}{x(x+h)}$$

Reduce the h factor

$$=-\frac{1}{x^2}$$

As h gets smaller and smaller, the denominator gets closer and closer to x^2 .