Evaluate the difference quotient $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = \sqrt{x}$.

This is a little different from $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$, but works the same way. Since a value is not supplied for $x$, we just leave it and work out the limit. Start by evaluating $f(x+h)$:

$$f(x+h) = \sqrt{x+h}$$

Now put this along with $f(x)$ into the difference quotient.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \cdot \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

Multiply the numerator and denominator by the conjugate of $\sqrt{x+h} - \sqrt{x}$

$$= \lim_{h \to 0} \frac{x+h-x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$\left(\sqrt{x+h} - \sqrt{x}\right)\left(\sqrt{x+h} + \sqrt{x}\right)$ is equal to $x + h - \sqrt{x+h}\sqrt{x} + \sqrt{x+h}\sqrt{x} - x$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

As $h$ gets smaller and smaller, the denominator gets closer and closer to $\sqrt{x} + \sqrt{x}$ or $2\sqrt{x}$ .