Evaluate the difference quotient $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-2x+4$.

This is a little different from $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, but works the same way. Since a value is not supplied for x, we just leave it and work out the limit. Start by evaluating $f(x+h)$:

$$\begin{aligned} f(x+h) &= (x+h)^2-2(x+h)+4 \\ &= x^2+2xh+h^2-2x-2h+4 \end{aligned}$$

Make sure you FOIL the square out and distribute the negative.

Now put this along with $f(x)$ into the difference quotient.

$$\begin{aligned} \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{\left(x^2+2xh+h^2-2x-2h+4\right)-\left(x^2-2x+4\right)}{h} \\ &= \lim_{h\to 0}\frac{2xh+h^2-2h}{h} \\ &= \lim_{h\to 0}\frac{h(2x+h-2)}{h} \\ &= \lim_{h\to 0}(2x+h-2) \end{aligned}$$

As h gets smaller and smaller, the term in the middle gets smaller. This means the limit is equal to $2x$ - 2. Since the other terms do not contain x, they are unaffected when h gets small.