

Find $\frac{d}{dx}[5^x]$.

This is just a basic application of the derivative rule for exponentials with $a = 5$:

$$\frac{d}{dx}[5^x] = \ln(5) \cdot 5^x$$

Find the derivative of $h(x) = x^2 e^x$.

Since this is a product

$$h(x) = \underset{u}{x^2} \underset{v}{e^x}$$

We'll use the product rule to find the derivative:

$$h'(x) = \underset{v}{e^x} \cdot \underset{u'}{2x} + \underset{u}{x^2} \cdot \underset{v'}{e^x}$$

This can be simplified by factoring e^x from each term:

$$h'(x) = e^x (2x + x^2)$$

Find the derivative of $y = \frac{2x^3 + x}{e^x}$.

Write this as a quotient

$$y = \frac{\underset{u}{2x^3 + x}}{\underset{v}{e^x}}$$

Now write out the quotient rule with the appropriate derivatives:

$$y' = \frac{\underset{v}{e^x} \underset{u'}{(6x^2 + 1)} - \underset{u}{(2x^3 + x)} \underset{v'}{e^x}}{\underset{v^2}{(e^x)^2}}$$

If we factor out the e^x on top, we can simplify this considerably:

$$y' = \frac{e^x(6x^2 + 1 - 2x^3 - x)}{(e^x)^2}$$
$$= \frac{6x^2 + 1 - 2x^3 - x}{e^x}$$

Find $\frac{d}{dx} [e^{5x^3+3x^2+x}]$

Start by writing this as a composition:

$$g(x) = 5x^3 + 3x^2 + x \rightarrow g'(x) = 15x^2 + 6x + 1$$
$$f(x) = e^x \rightarrow f'(x) = e^x$$

Applying the chain rule leaves us with

$$\frac{d}{dx} [e^{5x^3+3x^2+x}] = \underset{f'(g(x))}{e^{5x^3+3x^2+x}} \cdot \underset{g'(x)}{(15x^2 + 6x + 1)}$$