Section 3.1 Matrix Addition and Subtraction

Question 1 – How is information organized in a matrix?

Question 2 – How do you multiply a matrix by a scalar?

Question 3 - How do you add matrices?

Question 4 - How do you subtract matrices?

Question 1 – How is information organized in a matrix?

Key Terms

Size Transpose

Summary

A matrix is a table of numbers organized into rows and columns. Matrices are symbolized with capital letters. An example would be the matrix A below:

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 6 & 4 & 5 \end{bmatrix}$$

The entries in a matrix are placed within brackets. The size of a matrix is described by the number of rows and columns in the matrix. The matrix A above is a 2 × 3 matrix (read 2 by 3) since it has 2 rows and 3 columns. When quoting the size of a matrix, the number of rows are listed first and the number of columns is listed second.

Two matrices are equal if their corresponding entries (entries in the same row and column) are equal.

We can refer to the entries using lower case letters and subscripts. The notation a_{mn} is the entry in the matrix A in the mth row and nth column. In the matrix above, $a_{21} = 6$ is the entry in the second row and third column.

We can organize information in matrices by labeling the rows and column in a matrix. For instance, The Mundo Candy Company makes three types of chocolate candy: Cheery Cherry (CC), Mucho Mocha (MM), and Almond Delight (AD). Each kilogram of Cheery Cherry requires .5 kg of sugar and .2 kg of chocolate, each kilogram of Mucho Mocha requires .4 kg of sugar and .3 kg of chocolate; and each kilogram of Almond Delight requires .3 kg of sugar and .3 kg of chocolate.

Write a matrix which corresponds the type of candy with the amount of sugar and chocolate in the candy.

Start by deciding what size matrix to use. Since we are trying to match up 3 types of candy with 2 ingredients, we can use a 3×2 or a 2×3. For a 3×2 matrix, we label the rows with the type of candy and the columns with the ingredients:

$$\begin{array}{r} \\ \text{CC} \\ \text{MM} \\ \text{AD} \end{array} \begin{array}{c} \begin{array}{cc} \text{sugar} & \text{choc} \end{array} \\ \begin{bmatrix} .5 & .2 \\ .4 & .3 \\ .3 & .3 \end{bmatrix} \end{array}$$

For a 2×3, label the row with the ingredients and the columns with the types of candy,

$$\begin{array}{r} \\ \text{sugar} \\ \text{choco} \end{array} \begin{array}{c} \begin{array}{ccc} \text{CC} & \text{MM} & \text{AD} \end{array} \\ \begin{bmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{bmatrix} \end{array}$$

Either matrix corresponds the ingredients with the types of candy.

These two matrices are transposes of each other. This means that if we change the rows in one matrix to columns, we get the other matrix.

$$\begin{bmatrix} .5 & .2 \\ .4 & .3 \\ .3 & .3 \end{bmatrix} \xrightarrow[\text{SWITCH ROWS AND COLUMNS}]{\text{TRANSPOSE}} \begin{bmatrix} .5 & .4 & .3 \\ .2 & .3 & .3 \end{bmatrix}$$

<u>Notes</u>

Guided Example	Practice
Suppose you are given the matrix $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \\ -1 & 5 \end{bmatrix}$	1. Suppose you are given the matrix $B = \begin{bmatrix} -1 & 3 & -2 \\ 2 & 1 & -3 \\ 0 & 4 & 5 \end{bmatrix}$
a. Find the size of the matrix. **Solution** The matrix has three rows and two columns, so it is a 3 × 2 matrix.	a. Find the size of the matrix.
b. Find the value of a_{12} . **Solution** The entry a_{12} is the entry in the first row and second column so $a_{12} = -2$.	b. Find the value of b_{23} .
c. Find the value of a_{32} . **Solution** The entry a_{32} is the entry in the third row and second column so $a_{12} = 5$.	c. Find the value of b_{31} .

Guided Example	Practice
The Mundo Candy Company makes three types of chocolate candy: Cheery Cherry, Mucho Mocha, and Almond Delight. The company produces its products in San Diego, Mexico City, and Managua using two main ingredients: chocolate and sugar. The cost of 1 kg of sugar is \$4 in San Diego, \$2 in Mexico City, and \$1 in Managua. The cost of 1 kg of chocolate is \$3 in San Diego, \$5 in Mexico City, and \$7 in Managua. a. Find a 2 × 3 matrix that relates the cost of ingredients to the city. **Solution** Label the rows with the ingredients and the columns with the cities to give $$\begin{array}{c} \\ \text{sugar} \\ \text{choco} \end{array} \begin{array}{c} \begin{array}{ccc} SD & \text{MC} & M \end{array} \\ \begin{bmatrix} 4 & 2 & 1 \\ 3 & 4 & 7 \end{bmatrix} \end{array}$$ b. Find a 3 × 2 matrix that relates the cost of ingredients to the city. **Solution** Label the rows with the cities and the columns with the ingredients to give $$\begin{array}{c} \\ \text{SD} \\ \text{MC} \\ \text{M} \end{array} \begin{array}{c} \begin{array}{cc} \text{sugar} & \text{choco} \end{array} \\ \begin{bmatrix} 4 & 3 \\ 2 & 4 \\ 1 & 7 \end{bmatrix} \end{array}$$	2. The A-Plus auto parts store has two outlets, one in Phoenix and one in Tucson. Among other things, it sells wiper blades, air fresheners, and floor mats. During January, the Phoenix outlet sells 20 wiper blades, 10 air fresheners, and 8 floor mats. The Tucson outlet sells 15 wiper blades, 12 air fresheners, and 4 floor mats. During February, the Phoenix outlet sells 23 wiper blades, 8 air fresheners, and 4 floor mats. The Tucson outlet sells 12 wiper blades, 12 air fresheners, and 5 floor mats. a. Write a 2 x 3 matrix that reflects the January sales. Label the rows and columns of your matrix. b. Write a 2 x 3 matrix that February sales. Label the rows and columns of your matrix.

Question 2 – How do you multiply a matrix by a scalar?

Key Terms

Scalar

Summary

Multiplying a matrix by a scalar means multiplying the matrix by a number. To carry this multiplication out, multiply each entry in the matrix by the number.

Notes

Guided Example

For the matrix

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 3 & 4 \end{bmatrix}$$

Compute $3A$.

Solution Multiply each entry in the matrix to give

$$\begin{aligned} 3A &= 3\begin{bmatrix} 1 & 0 & -3 \\ 2 & 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3\cdot 1 & 3\cdot 0 & 3(-3) \\ 3\cdot 2 & 3\cdot 3 & 3\cdot 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 & -9 \\ 6 & 9 & 12 \end{bmatrix} \end{aligned}$$

Practice

1. For the matrix

$$B = \begin{bmatrix} 2 & 7 \\ 5 & -2 \end{bmatrix}$$

Compute $2B$.

Question 3 – How do you add matrices?

<u>Key Terms</u>

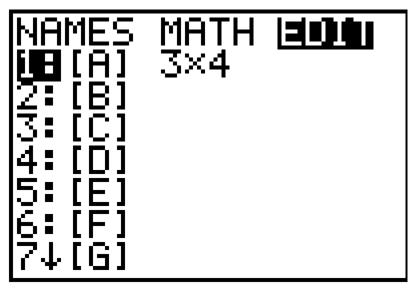
<u>Summary</u>

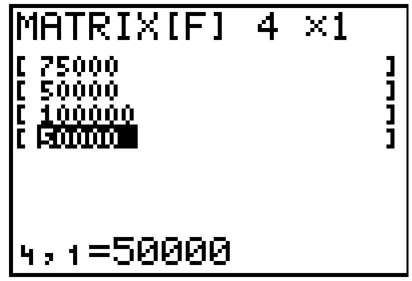
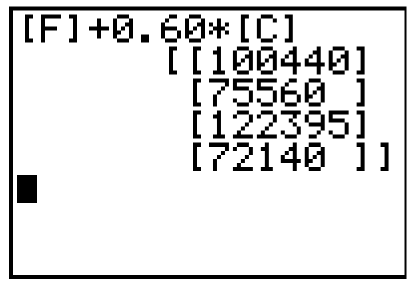
To be able to add two matrices, they must have the same size. If they do have the same size, they are added by adding the corresponding entries in the matrices.

Matrices can also be added, multiplied by a scalar ro subtracted on a TI graphing calculator. Suppose

$$C = \begin{bmatrix} 42400 \\ 42600 \\ 37325 \\ 36900 \end{bmatrix} \text{ and } F = \begin{bmatrix} 75000 \\ 50000 \\ 100000 \\ 50000 \end{bmatrix}$$

Find $F + 0.60C$ by following the steps below on a TI graphing calculator.

Enter the Matrix 1. Press [2nd] [x^{-1}] to access the MATRIX menu. 2. Use [▸] to go to EDIT. 3. Press [3] or move the cursor to 3: [C] and press [ENTER]. Note that if you used this matrix name before, it will have a dimension next to it.	
4. Enter the dimension of matrix C as 4 x 1. 5. Enter the values into the matrix as shown. Press [ENTER] after each entry. Note that the position is given at the bottom of the screen as 4,1=36900 etc. 6. Press [2nd] [MODE] to QUIT and return to the home screen.	

7. Repeat steps 1 through 5 to enter the matrix called [F].	
Perform the Matrix Arithmetic on the Home Screen 8. Press [2nd] [MODE] to QUIT and return to the Home screen. 9. You must use MATRIX NAMES to enter the names of the matrices. Press [2nd] [x^{-1}] to access NAMES. 10. Select 6: [F] and press [ENTER]. 11. Use the operation key [+] to add. Follow this command with [0][.][6][0][×] 12. Repeat steps 9 and 10 the process to select matrix 3:[C] and press [ENTER]. 13. To carry out the operations, press [ENTER].	

Notes

Guided Example	Practice
Suppose $A = \begin{bmatrix} 0 & -2 \\ 3 & 1 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & -2 \\ 0 & -2 \end{bmatrix}$. a. Compute $A + B$. **Solution** Both matrices have size 3 × 2 so they may be added. Add the corresponding entries to give $$A+B = \begin{bmatrix} 0+3 & -2+1 \\ 3+2 & 1+(-2) \\ -1+0 & 1+(-2) \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 5 & -1 \\ -1 & -1 \end{bmatrix}$$ b. Compute $2A + 5B$ **Solution** To make it easier to carry out the matrix addition, first multiply the scalars and matrices. Then add the corresponding entries: $$2A+5B = \begin{bmatrix} 0 & -4 \\ 6 & 2 \\ -2 & 8 \end{bmatrix} + \begin{bmatrix} 15 & 5 \\ 10 & -10 \\ 0 & -10 \end{bmatrix} = \begin{bmatrix} 0+15 & -4+5 \\ 6+10 & 2+(-10) \\ -2+0 & 8+(-10) \end{bmatrix} = \begin{bmatrix} 15 & 1 \\ 16 & -8 \\ -2 & -2 \end{bmatrix}$$	1. Suppose $C = \begin{bmatrix} 7 & -2 \\ 1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$. a. Compute $C + D$. b. Compute $3C + 4D$

Question 4 – How do you subtract matrices?

Key Terms

Summary

To be able to subtract two matrices, they must have the same size. If they do have the same size, they are subtracted by subtracting the corresponding entries in the matrices.

Notes

Guided Example	Practice
Suppose $A = \begin{bmatrix} 0 & -2 \\ 3 & 1 \\ -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & -2 \\ 0 & -2 \end{bmatrix}$. a. Compute *A* - *B*. **Solution** The sizes of the matrices match so we can subtract corresponding entries, $A - B = \begin{bmatrix} 0-3 & -2-1 \\ 3-2 & 1-(-2) \\ -1-0 & 4-(-2) \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ 1 & 3 \\ -1 & 6 \end{bmatrix}$ b. Compute $\frac{1}{2}A - B$ **Solution** Multiply A by the scalar and then subtract corresponding entries, $\frac{1}{2}A - B = \begin{bmatrix} 0 & -1 \\ \frac{3}{2} & \frac{1}{2} \\ -\frac{1}{2} & 2 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & -2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-3 & -1-1 \\ \frac{3}{2}-2 & \frac{1}{2}-(-2) \\ -\frac{1}{2}-0 & 2-(-2) \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -\frac{1}{2} & \frac{5}{2} \\ -\frac{1}{2} & 4 \end{bmatrix}$	1. Suppose $C = \begin{bmatrix} 7 & -2 \\ 1 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$. a. Compute *C* - *D*. b. Compute $2C - 0.5D$